MATH 1030: Unit 4 Review B **Name:** Key

1- Determine the probability of the following event.
What is the probability that the next person you meet was not born in July? (Assume 365 days in a year)

$334/365$

2- Find both the odds for and the odds against the following events.
Flipping two fair coins and getting a head and a tail.

HT
TH
HH
TT

odds for $\frac{2}{2} =$ 1 to 1

odds against $\frac{2}{2} =$ 1 to 1

3- Decide which method (theoretical, relative frequency, or subjective) is appropriate, and compute or estimate the following probability. Randomly selecting a pair of white socks from a drawer that holds five pairs of black socks, nine pairs of blue socks, and eight pairs of white socks.

$\frac{16}{44} = \frac{4}{11}$

4- Determine whether the following events are independent or dependent. Then find the probability of the event. Drawing three queens in a row from a standard deck of cards when the drawn card is returned to the deck each time.

$\frac{1}{13} \cdot \frac{1}{13} \cdot \frac{1}{13} = \frac{1}{2197}$

5- Randomly selecting a three-person committee consisting entirely of Americans from a pool of 12 British people and 18 Americans.

$\frac{{}_{18}C_3}{{}_{30}C_3} = \frac{816}{4060} = \frac{204}{1015}$

6- Determine whether the following events are overlapping or non-overlapping. Then find the probability of the event. Drawing either a jack or a spade from a regular deck of cards.

$\left(\frac{1}{13} + \frac{1}{4}\right) - \frac{1}{52} = \frac{4}{13}$

7- Use the method of your choice to determine the following probability:
Randomly selecting a girl or a non-soccer player from a sixth-grade class of 12 boys, 7 of whom play soccer, and 15 girls, 10 of whom play soccer.

$$\left(\frac{15}{27}+\frac{10}{27}\right)-\frac{5}{27}=\frac{20}{27}$$

8- Find the expected value (to you) of the described game. You are given 9 to 1 odds against tossing three heads with three coins, meaning you win \$9 if you succeed and you lose \$1 if you fail.

$$\left(\frac{1}{8}\cdot \$9\right)+\left(\frac{7}{8}\cdot(-\$1)\right)=\frac{1}{4}=0.25$$

9- Find the expected value (to the company) per policy sold. If the company sells 10,000 policies, what is the expected profit or loss? An insurance policy sells for \$400. Based on past data, an average of 1 in 50 policyholders will file a \$4,000 claim, an average of 1 in 100 policyholders will file a \$8,000 claim, and an average of 1 in 200 policyholders will file a \$20,000 claim.

$$(400\times 1)+\left(-4{,}000\cdot\frac{1}{50}\right)+\left(-8{,}000\cdot\frac{1}{100}\right)+\left(-30{,}000\cdot\frac{1}{200}\right)$$
$= \$90$ expected value per policy.

$\$90 \cdot 10{,}000 = \$900{,}000$ expected profit.

Answer the following questions using the appropriate counting technique, which may be either arrangements with repetition, permutations, or combinations.

10- A city council with ten members must appoint a four-person subcommittee. How many subcommittees are possible?

$${}_{10}C_4 = 210$$

11- A grand opening of an ice cream parlor is giving away free ice cream. They have 20 flavors and you want a triple scoop ☺. How many different triple scoop ice creams can you have? [illegible]

$${}_{20}C_3 = 1{,}140$$

Find the probability of the given event.

12- Guessing the top three winners (in order) from a group of eight finalists in a soccer tournament.

$$\frac{1}{{}_8P_3}=\frac{1}{336}$$

State whether the growth (or decay) is linear or exponential, and answer the associated question.

13- The price of gold is increasing \$0.90 per week. If the price is \$1320 per ounce today, what will it be in ten weeks?

$$f(p) = .90\cancel{p} + 1320$$

$$.90(10) + 1320 = \$1329$$

14- The value of your car is decreasing by 10% per year. If the car is worth \$25,000 today, what will it be worth in ten years?

$$25{,}000(1-.10)^{10} = \$8716.96$$

15- The following exercise gives a doubling time for an exponentially growing quantity. The initial population of a town is 12,000, and it grows with a doubling time of 8 years. What will the population be in 12 years? In 24 years?

$$12{,}000(2)^{\frac{t}{8}}$$

12 years about 34,000

24 years about 96,000

Use the approximate doubling time formula (rule of 70).

Discuss whether the formula is valid for the case described. — yes because it's under 15%.

16- A city's population is growing at a rate of 3.5% per year. What is its doubling time? By what factor will the population increase in 50 years?

$$T_D \approx \frac{70}{P} = \frac{70}{3.5} = 20 \text{ years}$$

$$X\left(2^{\frac{50}{20}}\right) = X(5.66)$$ a factor of 5.66

Each exercise gives a half-life for an exponentially decaying quantity. Answer the questions that follow.

17- Radium-226 is a metal with a half-life of 1600 years. If you start with 1 kilogram of radium-226, how much will remain after 1000 years? After 10,000 years?

1,000 years ; $1\left(\frac{1}{2}\right)^{\frac{1000}{1600}} \approx .65$ kgs

10,000 years ; $1\left(\frac{1}{2}\right)^{\frac{10,000}{1600}} \approx 0.013$ kgs yes it's below 15%

Use the approximate half-life formula. Discuss whether the formula is valid for the case described.

18- The production of a gold mine decreases 5% per year. What is the approximate half-life for the production decline? If its current annual production is 5000 kilograms, about what will its production be in 10 years?

$$T_{half} = \frac{70}{P} = \frac{70}{5} = 14 \text{ years}$$

10 years: $5{,}000\left(\frac{1}{2}\right)^{\frac{10}{14}} \approx 3047.53$

about 3050 kg.

Starting from a 2013 population of 7.1 billion, use the given growth rate to find the approximate doubling time (use the rule of 70) and to predict world population in 2050.

19- Use the current annual growth rate of the United States, which is about 0.7%

$$T_D = \frac{70}{7.1} \approx 98.59 \text{ or about } 100 \text{ years}$$

in 2050: $7.1(2)^{\frac{37}{100}} \approx 9.175$

or about 9.2 billion

Use the earthquake magnitude scale to answer the question:

20- How much energy, in joules, was released by the 1962 earthquake in Cache Valley in Utah (magnitude 5.5)? This is one of the most powerful earthquake ever recorded in Utah history.

$$E = 2.5 \times 10^4 \times 10^{1.5M}$$

$$= 2.5 \times 10^4 \times 10^{1.5(5.5)}$$

$$2.5 \times 10^4 \times 10^{8.25}$$

$$\cancel{2.5 \times 10^{12.25}}$$

$$\approx 4.4 \times 10^{12}$$

or about 4.4 billion

21

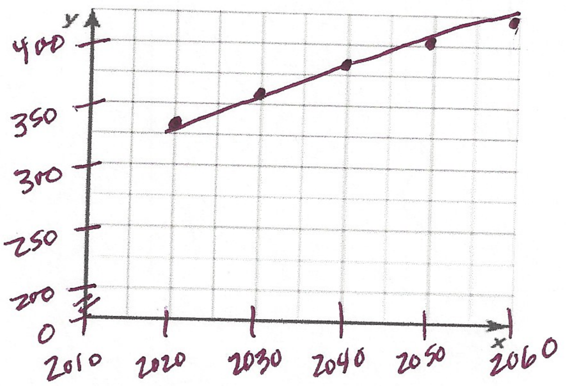
~~22~~- The following data table represents a function.

a) Identify the independent and dependent variables, and describe the domain and range.

Year	Projected U.S. Population (millions)
2020	334
2030	358
2040	380
2050	400
2060	420

Independent : time
dependent : population
Domain : years between 2020 and 2060
Range : population between 334 and 420 million

b) Make a clear graph of the function (label the x and y-axis). Describe the function in words.



The projected population increases steadily with time.

22

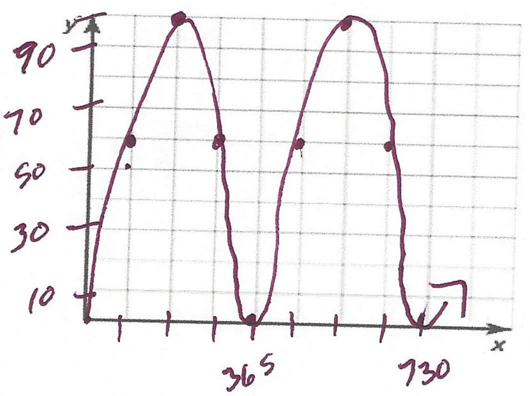
~~23~~- Answer the following to describe a possible relationship between the variables.

(Day of year, high temperature) over a two-year period for the town in which you are living.

a) Describe an **appropriate** domain and range for the function.

Domain: days over two year period (Could be 1-730)
Range: temperatures 0° to 100° (for Utah)

b) Make a rough sketch of a graph of the function (label the x and y-axis). Explain the assumptions that go into your sketch.



January 1st is #1
then split the year into 4 equal spaces.
Graph shows average temperature for each day.

The following situations can be modeled by linear functions. In each case, write an equation for the linear function and use it to answer the given question. Be sure you clearly identify the independent and dependent variables.

~~24~~- In 2000, the population of Boom Town began increasing at a rate of 300 people per year. The population in 2000 was 1500 people. What is your projection for the population in the year 2050?
23

Variables are time and population (t, p)
t is years, $t=0$ at year 2000.

$P = 1500 + 300t$
$= 1500 + 300(50) = 16,500$

Create the required linear function, and use it to answer the following questions.

~~25~~- 24 You can purchase a motorcycle for $6,500 or lease it for a down payment of $200 and $150 per month. Find a function that describes how the cost of the lease depends on time. How long can you lease the motorcycle before you've paid more than its purchase price?

$f(x) = 150x + 200$

$6500 = 150x + 200$

$x = 42$ months

~~26~~- 25 Consider the following cases of exponential growth and decay.
A certain drug breaks down in the human body at a rate of 15% per hour. The initial amount of the drug in the bloodstream is 8 milligrams.

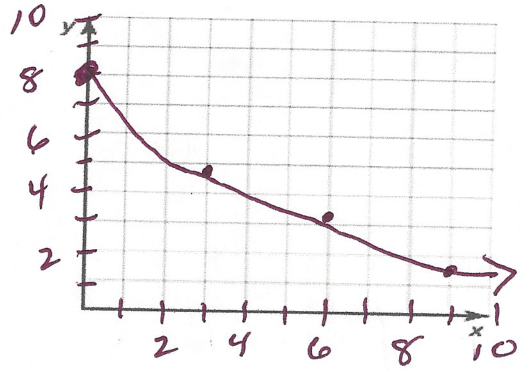
a) Create an exponential function of the form $Q = Q_0 \times (1 + r)^t$ (where $r > 0$ for growth and $r < 0$ for decay) to model the situation described. Be sure to clearly identify both variables in your function.

$Q = 8 \times (0.85)^t$

b) Create a table showing the value of the quantity Q for at least 4 units of time (either years, months, weeks, or hours) of growth or decay.

hours	drug
0	8
3	4.91
6	3.02
9	1.85

c) Make a graph of the exponential function (label the x and y-axis).



~~27~~- 26 Assume that for the average individual, Dextromethorphan (DXM), a common ingredient in cough syrup, has a half-life of 3 hours in the bloodstream. At 6:00 a.m., you take a 400-milligram dose of DXM.

a) How much DXM will be in your blood at 10:00 a.m. the same day?
At 6:00 p.m. that evening?

~~b) Estimate when the amount of DXM will decay to 20% of its original amount.~~

10 a.m.: $400\left(\frac{1}{2}\right)^{\frac{4}{3}}$ about 159 mg.

6 p.m.: $400\left(\frac{1}{2}\right)^{\frac{12}{3}}$ about 25 mg.