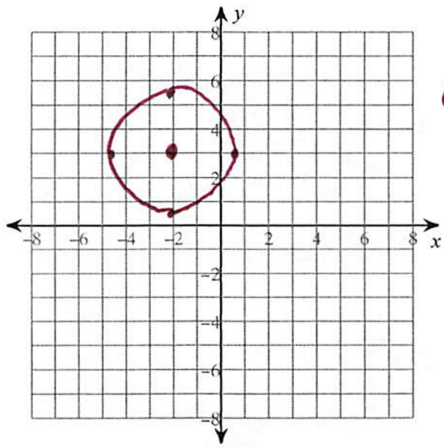
Pre-Calculus Name Key

Circles and Ellipses

Date________________ Period____

Identify the center and radius of each. Then sketch the graph.

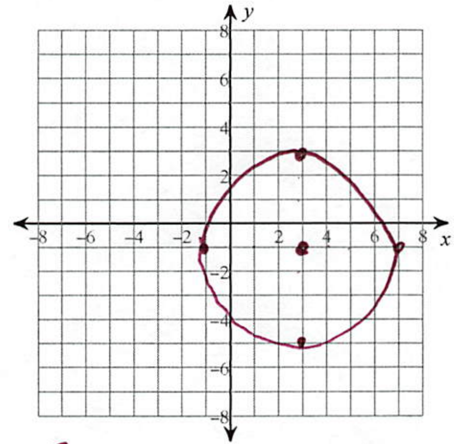
1) $(x+2)^2+(y-3)^2=8$



$c(-2,3)$

$r=2\sqrt{2}$

2) $x^2+y^2-6x+2y-6=0$



$c(3,-1)$

$r=4$

$x^2-6x \quad +y^2+2y \quad = 6 +9+1$

$(x^2-6x+9)+(y^2+2y+1)=16$

$(x-3)^2+(y+1)^2=16$

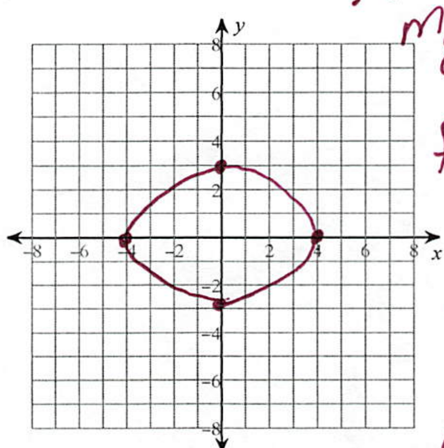
Graph the ellipse:
Identify the center, all 4 vertices (major and minor), foci, eccentricity and area.

3) $\dfrac{x^2}{16}+\dfrac{y^2}{9}=1$

$M_j\ (4,0)\ (-4,0)$

$M_i\ (0,3)(0,-3)$

$c(0,0)$



$f(\sqrt{7},0)$
$(-\sqrt{7},0)$

$e=\dfrac{\sqrt{7}}{4}$

$A=12\pi u^2$

$c^2=a^2-b^2$

$16-9$

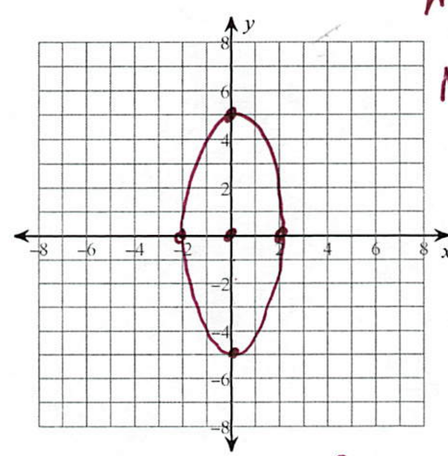
$c^2=7$

4) $\dfrac{x^2}{4}+\dfrac{y^2}{25}=1$

$c(0,0)$

$M_j\ (0,5)(0,-5)$

$M_i\ (0,2)(0,-2)$



Foci $(0,\sqrt{21})$
$(0,-\sqrt{21})$

$e=\dfrac{\sqrt{21}}{5}$

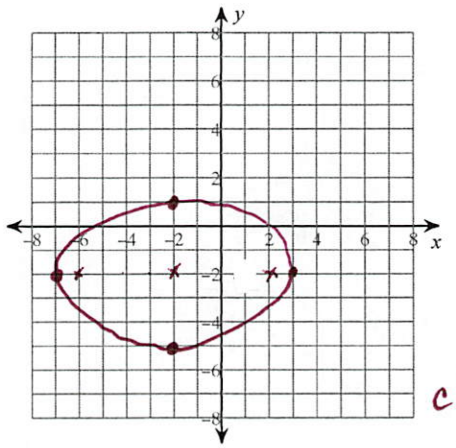
$A=10\pi u^2$

$c^2=a^2-b^2$

$25-4$

$=21$

5) $\frac{(x+2)^2}{25} + \frac{(y+2)^2}{9} = 1$



$c^2 = 25 - 9$

$c = 4$

$C(-2,-2)$

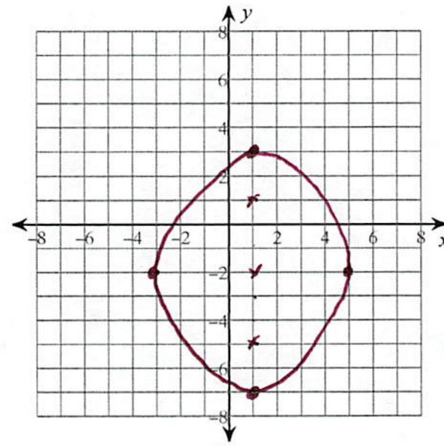
$M_j\ (3,-2)(-7,-2)$

$M_i\ (-2,1)(-2,-5)$

Foci $(2,-2)(-6,-2)$

$e = \frac{4}{5}$ $A = 15u^2$

6) $\frac{(x-1)^2}{16} + \frac{(y+2)^2}{25} = 1$



$c^2 = 25 - 16$

$c = 3$

$C(1,-2)$

$M_j\ (1,3)(1,-7)$

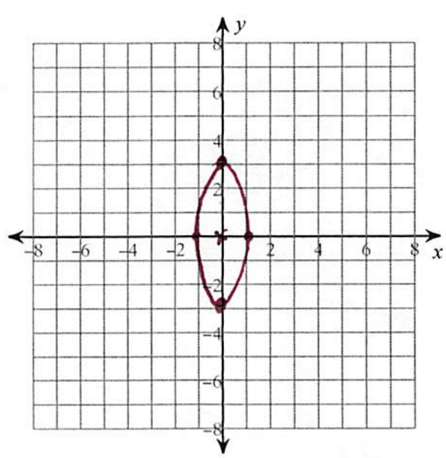
$M_i\ (5,-2)(-3,-2)$

Foci $(1,1)(1,-5)$

$e = \frac{3}{5}$ $A = 20u^2$

Rewrite the equation in standard form and graph:
Identify the center, all 4 vertices (major and minor), foci, eccentricity and area.

7) $9x^2 + y^2 - 9 = 0$



$9x^2 + y^2 = 9$

$x^2 + \frac{y^2}{9} = 1$

$C(0,0)$

$M_j(0,3)(0,-3)$

$M_i\ (1,0)(-1,0)$

foci $(0,2\sqrt{2})(0,-2\sqrt{2})$

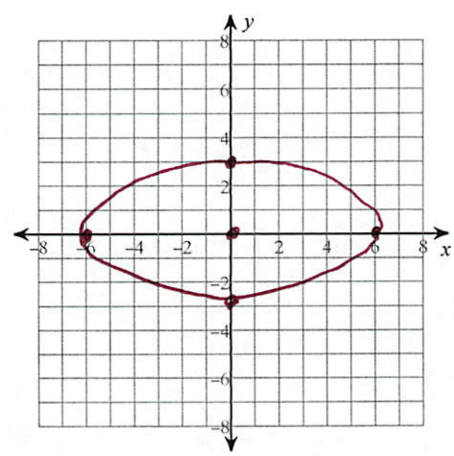
$c^2 = 9 - 1$

$c = \sqrt{8} = 2\sqrt{2}$

$e = \frac{2\sqrt{2}}{3}$

$A = 3u^2$

8) $x^2 + 4y^2 - 36 = 0$



$C(0,0)$

$M_j\ (6,0)(-6,0)$

$M_i\ (0,3)(0,-3)$

Foci $(3\sqrt{3},0)(-3\sqrt{3},0)$

$e = \frac{3\sqrt{3}}{6}$

$= \frac{\sqrt{3}}{2}$

$A = 18\pi u^2$

$x^2 + 4y^2 = 36$

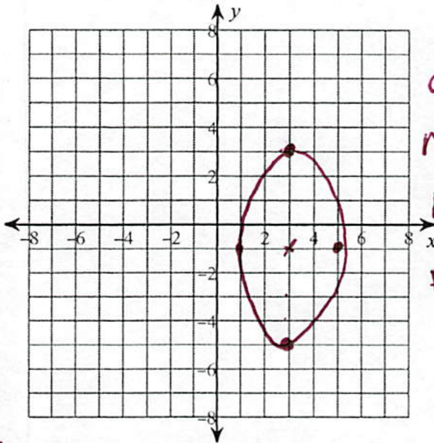
$\frac{x^2}{36} + \frac{y^2}{9} = 1$

$c^2 = 36 - 9$

$c = \sqrt{27} = 3\sqrt{3}$

Identify the vertices and foci of each. Then sketch the graph.

9) $4x^2 + y^2 - 24x + 2y + 21 = 0$



$c(3,-1)$

$M_j\ (3,3)(3,-5)$

$M_i\ (5,-1)(1,-1)$

Foci $(3, 2\sqrt{3}-1)$
$(3, -2\sqrt{3}-1)$

$e = \frac{2\sqrt{3}}{4} = \frac{\sqrt{3}}{2}$

$A = 8\pi u^2$

$4x^2 - 24x \quad y^2 + 2y = -21$

$4(x^2 - 6x + 9) + (y^2 + 2y + 1) = -21 + 36 + 1$

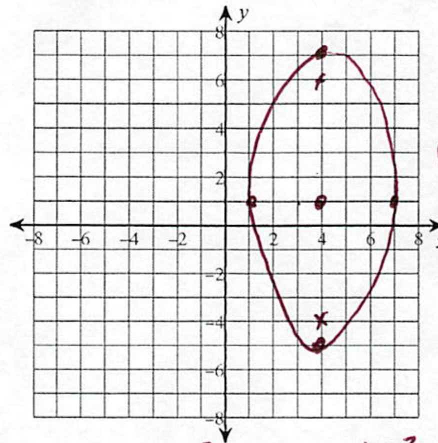
$4(x-3)^2 + (y+1)^2 = 16$

$$\boxed{\frac{(x-3)^2}{4} + \frac{(y+1)^2}{16} = 1}$$

$c^2 = 16 - 4$

$c = \sqrt{12} = 2\sqrt{3}$

10) $4x^2 + y^2 - 32x - 2y + 29 = 0$



$c(4,1)$

$M_j\ (4,7)(4,-5)$

$M_i\ (7,1)(1,1)$

Foci $(4,6)(4,-4)$

$e = \frac{5}{6}$ $\quad A = 18\pi u^2$

$4x^2 - 32x \quad + y^2 - 2y = -29$

$4(x^2 - 8x + 16) + (y^2 - 2y + 1) = -29 + 64 + 1$

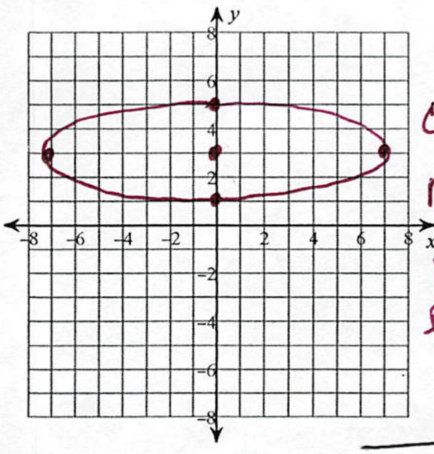
$4(x-4)^2 + (y-1)^2 = 36$

$$\boxed{\frac{(x-4)^2}{9} + \frac{(y-1)^2}{36} = 1}$$

$c^2 = 36 - 9$

$c = 5$

11) $4x^2 + 49y^2 - 294y + 245 = 0$



$c(0,3)$

$M_j\ (7,3)(-7,3)$

$M_i\ (0,5)(0,1)$

Foci $(3\sqrt{5},3)$
$(-3\sqrt{3},3)$

$e = \frac{3\sqrt{5}}{7}$ $\quad A = 14\pi u^2$

$4x^2 \quad + 49y^2 - 294y = -245$

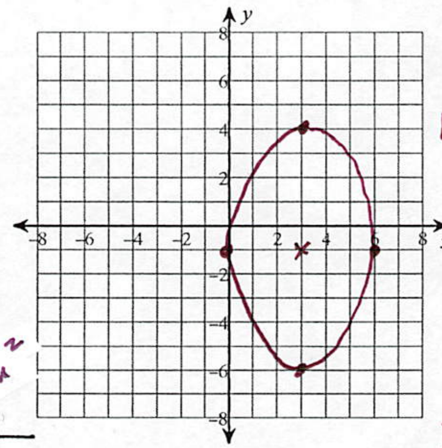
$4x^2 + 49(y^2 - 6y + 9) = -245 + 441$

$4x^2 + 49(y-3)^2 = 196$

$$\boxed{\frac{x^2}{49} + \frac{(y-3)^2}{4} = 1}$$

$c^2 = 49 - 4 = 45$ $\quad c = \sqrt{45} = 3\sqrt{5}$

12) $25x^2 + 9y^2 - 150x + 18y + 9 = 0$



$C(3,-1)$

$M_j\ (3,4)(3,-6)$

$M_i\ (6,-1)(0,-1)$

Foci $(3,3)(3,-5)$

$e = \frac{4}{5}$

$A = 15\pi u^2$

$25x^2 - 150x \quad 9y^2 + 18y = -9$

$25(x^2 - 6x + 9) + 9(y^2 + 2y + 1) = -9 + 225 + 9$

$25(x-3)^2 + 9(y+1)^2 = 225$

$\frac{(x-3)^2}{9} + \frac{(y+1)^2}{25} = 1$

$c^2 = 25 - 9$

$c = 4$

Use the information provided to write the standard form equation of each ellipse.

13) Major-Vertices: $(0, 5)$, $(0, -5)$ $\updownarrow$
Foci: $(0, 4)$, $(0, -4)$

$C(0,0)$
$a = 5$
$c = 4$

$c^2 = a^2 - b^2$
$16 = 25 - b^2$
$b^2 = 9$

$$\boxed{\frac{x^2}{9} + \frac{y^2}{25} = 1}$$

14) Major-Vertices: $(5, 0)$, $(-5, 0)$ $\longleftrightarrow$
Foci: $(3, 0)$, $(-3, 0)$

$C(0,0)$
$a = 5$
$c = 3$

$9 = 25 - b^2$
$b^2 = 16$

$$\boxed{\frac{x^2}{25} + \frac{y^2}{16} = 1}$$

15) Major-Vertices: $(11, -9)$, $(1, -9)$ $\longleftrightarrow$
Foci: $(9, -9)$, $(3, -9)$

$C(6, -9)$
$a = 5$
$c = 3$

$9 = 25 - b^2$
$b^2 = 16$

$$\boxed{\frac{(x-6)^2}{25} + \frac{(y+9)^2}{16} = 1}$$

16) Major-Vertices: $(3, 16)$, $(3, -10)$ $\updownarrow$
Foci: $(3, 8)$, $(3, -2)$ $C(3,3)$

$a = 13$
$c = 5$

$25 = 169 - b^2$
$b^2 = 144$

$$\boxed{\frac{(x-3)^2}{169} + \frac{(y-3)^2}{144} = 1}$$

17) Foci: $(7 + 4\sqrt{6}, -2)$, $(7 - 4\sqrt{6}, -2)$
Minor-vertices: $(7, 3)$, $(7, -7)$

$C(7, -2)$
$b = 5$
$c = 4\sqrt{6}$

$(4\sqrt{6})^2 = a^2 - 5^2$
$a^2 = 121$
$a = 11$

$$\boxed{\frac{(x-7)^2}{121} + \frac{(y+2)^2}{25} = 1}$$

18) Foci: $(-8, -6 + 3\sqrt{5})$, $(-8, -6 - 3\sqrt{5})$
Minor-vertices: $(-2, -6)$, $(-14, -6)$

$C(-8, -6)$
$c = 3\sqrt{5}$
$b = 6$

$(3\sqrt{5})^2 = a^2 - 6^2$
$a^2 = 81$

$$\boxed{\frac{(x+8)^2}{36} + \frac{(y+6)^2}{81} = 1}$$

19) Foci: $(5 + 2\sqrt{6}, 9)$, $(5 - 2\sqrt{6}, 9)$
Endpoints of major axis: $(12, 9)$, $(-2, 9)$

$C(5, 9)$
$c = 2\sqrt{6}$

$2a = 14$
$a = 7$

$(2\sqrt{6})^2 = 7^2 - b^2$
$b^2 = 25$

$$\boxed{\frac{(x-5)^2}{49} + \frac{(y-9)^2}{25} = 1}$$

20) Foci: $(8, -5 + 4\sqrt{6})$, $(8, -5 - 4\sqrt{6})$
Endpoints of minor axis: $(13, -5)$, $(3, -5)$

$C(8, -5)$
$c = 4\sqrt{6}$

$2b = 10$
$b = 5$

$(4\sqrt{6})^2 = a^2 - 5^2$
$a^2 = 121$

$$\boxed{\frac{(x-8)^2}{25} + \frac{(y+5)^2}{121} = 1}$$

21) Center: $(7, -9)$
Major Vertex: $(-4, -9)$
Focus: $(7 + \sqrt{85}, -9)$

$a = 11$
$c = \sqrt{85}$

$(\sqrt{85})^2 = 11^2 - b^2$
$b^2 = 36$

$$\boxed{\frac{(x-7)^2}{121} + \frac{(y+9)^2}{36} = 1}$$

22) Center: $(-7, 4)$
Minor-Vertex: $(2, 4)$
Focus: $(-7, 4 - \sqrt{19})$

$b = 9$
$c = \sqrt{19}$

$(\sqrt{19})^2 = a^2 - 9^2$
$a^2 = 100$

$$\boxed{\frac{(x+7)^2}{100} + \frac{(y-4)^2}{81} = 1}$$