Precalculus
Chapter 7 Review

Name: ____________________
Period: ______

Show all of your work!

1. Determine the order of the matrix:

1. $\begin{bmatrix} 5 & -1 & 6 \\ -8 & 0 & 3 \\ 3 & 1 & 9 \end{bmatrix}$

2. $\begin{bmatrix} 10 & -2 & 8 & 0 \\ 3 & 11 & 18 & -4 \end{bmatrix}$

3. $\begin{bmatrix} 6 & 7 & -4 \end{bmatrix}$

Identify the element specified in the matrix in problem #1

4. a_{23}

5. a_{31}

6. a_{22}

7. a_{11}

Use the matrices to answer questions 8-13:

$A = \begin{bmatrix} 2 & 3 \\ -1 & 5 \end{bmatrix}$ $B = \begin{bmatrix} 1 & -3 \\ -2 & -4 \end{bmatrix}$ $C = \begin{bmatrix} 1 & 3 & -2 \\ 2 & -4 & -1 \end{bmatrix}$ $D = \begin{bmatrix} 3 & -5 \\ -2 & 0 \\ 4 & 1 \end{bmatrix}$

8- What is $2A - 4B$?

9- What is $|A|$?

10- Find B^{-1} ?

11- Find AC

12. Find CA

13- Find CD

Find the determinant of each matrix:

14. $\begin{bmatrix} 3 & 6 \\ -2 & -4 \end{bmatrix}$

15. $\begin{bmatrix} 1 & 2 & -1 \\ 2 & -1 & 3 \\ 3 & 1 & -2 \end{bmatrix}$

16. $\begin{bmatrix} 5 & -2 \\ 6 & -1 \end{bmatrix}$

Solve for the variable:

17. $\begin{bmatrix} 2 & x-1 \\ 2 & 3 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 2 & -3 \\ y+2 & 3 \\ -z & 2 \end{bmatrix}$

Write the augmented matrix that represents the system:

18. $\begin{cases} x - 2y + z = 7 \\ 3x - 5y + z = 14 \\ 2x - 2y - z = 3 \end{cases}$

Write the augmented matrices in reduced row echelon form:

19. $\begin{bmatrix} 4 & 5 & -24 \\ 2 & -3 & 0 \end{bmatrix}$

20. $\begin{bmatrix} 1 & -2 & 1 & 7 \\ 0 & 1 & -2 & -7 \\ 2 & -2 & -3 & 3 \end{bmatrix}$

Write the augmented matrices in reduced row echelon form:

21. $\begin{bmatrix} 1 & -1 & 1 & 0 \\ 2 & 0 & -3 & -1 \\ -1 & -1 & 2 & -1 \end{bmatrix}$

22. Verify that the matrices are inverses of each other:

$$A = \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}, \quad B = \begin{bmatrix} 0.8 & -0.2 \\ -0.6 & 0.4 \end{bmatrix}$$

Write a matrix equation for the system, and then solve using inverse matrices:

23. $\begin{cases} 3x - 2y = 0 \\ -x + y = 5 \end{cases}$

Write a matrix equation for the system, and then solve using inverse matrices:
(Use a Calculator to find the inverse)

24. $\begin{cases} 3x - 3y + 6z = 20 \\ x - 3y + 10z = 40 \\ -x + 3y - 5z = 30 \end{cases}$

Write the matrix in row echelon form to solve the system:

25. $\begin{cases} x + 2y - 3z = -7 \\ 2x - 3y + z = 14 \\ 4x + y - 2z = 3 \end{cases}$

26. $\begin{cases} 2x + 3y - 12z = 1 \\ x - 2y + z = 4 \\ 4x + y - 14z = 7 \end{cases}$

Use cofactor and minors to find the determinant of the matrices.
Determine whether the matrix has an inverse. **(But don't find the inverse).**

27. $\begin{vmatrix} 0 & -1 & 0 \\ 2 & 6 & 4 \\ 1 & 0 & 3 \end{vmatrix}$

Use a calculator for #28 ☺

28. $\begin{vmatrix} 1 & 3 & 3 & 0 \\ 0 & 2 & 0 & 1 \\ -1 & 0 & 0 & 2 \\ 1 & 6 & 4 & 1 \end{vmatrix}$

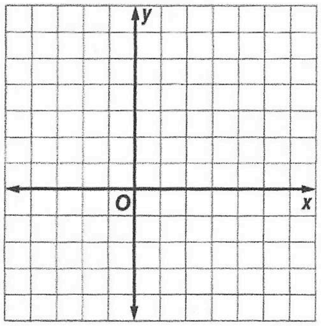
Use Cramer's Rule to solve the system:

29. $\begin{cases} 6x + 12y = 33 \\ 4x + 7y = 20 \end{cases}$

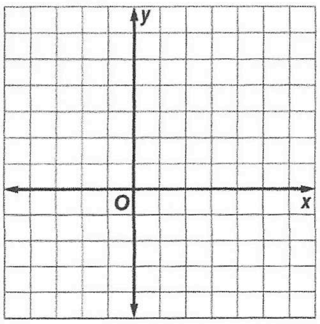
30. $\begin{cases} x + 2y + 2z = 6 \\ x - 2y = -1 \\ 2x + y + 3z = 7 \end{cases}$

Sketch the triangle with the given vertices and use cofactor expansion to find its area:

31- $(1, 0)$ $(3, 5)$ $(-2, 2)$



32- $(-2, 5)$ $(7, 2)$ $(3, 4)$



Precalculus
Chapter 7 Review

Name: Key
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Show all of your work!

1. Determine the order of the matrix:

1. $\begin{bmatrix}5 & -1 & 6\\-8 & 0 & 3\\3 & 1 & 9\end{bmatrix}$ $\boxed{3\times 3}$

2. $\begin{bmatrix}10 & -2 & 8 & 0\\3 & 11 & 18 & -4\end{bmatrix}$ $\boxed{2\times 4}$

3. $\begin{bmatrix}6 & 7 & -4\end{bmatrix}$ $\boxed{1\times 3}$

Identify the element specified in the matrix in problem #1

4. a_{23} $\boxed{3}$

5. a_{31} $\boxed{3}$

6. a_{22} $\boxed{0}$

7. a_{11} $\boxed{5}$

Use the matrices to answer questions 8-13:

$A = \begin{bmatrix}2 & 3\\-1 & 5\end{bmatrix}$ $B = \begin{bmatrix}1 & -3\\-2 & -4\end{bmatrix}$ $C = \begin{bmatrix}1 & 3 & -2\\2 & -4 & -1\end{bmatrix}$ $D = \begin{bmatrix}3 & -5\\-2 & 0\\4 & 1\end{bmatrix}$

8- What is $2A - 4B$?

$2\begin{bmatrix}2 & 3\\-1 & 5\end{bmatrix} - 4\begin{bmatrix}1 & -3\\-2 & -4\end{bmatrix}$

$\begin{bmatrix}4 & 6\\-2 & 10\end{bmatrix} + \begin{bmatrix}-4 & 12\\8 & 16\end{bmatrix} = \boxed{\begin{bmatrix}0 & 18\\6 & 26\end{bmatrix}}$

9- What is $|A|$?

$10 - (-3) = \boxed{13}$

10- Find B^{-1}?

$|B| = -4 - (-6) = -10$

$-\frac{1}{10}\begin{bmatrix}-4 & 3\\2 & 1\end{bmatrix} = \boxed{\begin{bmatrix}2/5 & -3/10\\-1/5 & -1/10\end{bmatrix}}$

11- Find AC

$\begin{bmatrix}2 & 3\\-1 & 5\end{bmatrix} \cdot \begin{bmatrix}1 & 3 & -2\\2 & -4 & -1\end{bmatrix}$ $\cancel{2}\ -1$ $\cancel{3}\ 5$ $= \boxed{\begin{bmatrix}8 & -6 & -7\\9 & -23 & -3\end{bmatrix}}$

12. Find CA

$\begin{bmatrix}1 & 3 & -2\\2 & -4 & -1\end{bmatrix} \cdot \begin{bmatrix}2 & 3\\-1 & 5\end{bmatrix}$

$\boxed{\text{Not Possible}}$

13- Find CD

$\begin{bmatrix}1 & 3 & -2\\2 & -4 & -1\end{bmatrix} \cdot \begin{bmatrix}3 & -5\\-2 & 0\\4 & 1\end{bmatrix}$ $\cancel{1}\ 2$ $\cancel{3}\ -4$ $\cancel{-2}\ -1$

$= \boxed{\begin{bmatrix}-11 & -7\\10 & -11\end{bmatrix}}$

Find the determinant of each matrix:

14. $\begin{bmatrix} 3 & 6 \\ -2 & -4 \end{bmatrix}$

$-12-(-12)$

$=\boxed{0}$

15. $\begin{bmatrix} 1 & 2 & -1 \\ 2 & -1 & 3 \\ 3 & 1 & -2 \end{bmatrix}$

$1\begin{vmatrix} -1 & 3 \\ 1 & -2 \end{vmatrix} -2\begin{vmatrix} 2 & 3 \\ 3 & -2 \end{vmatrix} + -1\begin{vmatrix} 2 & -1 \\ 3 & 1 \end{vmatrix}$

$1(-1)-2(-13)-1(5)$

$=\boxed{20}$

16. $\begin{bmatrix} 5 & -2 \\ 6 & -1 \end{bmatrix}$

$-5-(-12)$

$=\boxed{7}$

Solve for the variable:

17. $\begin{bmatrix} 2 & x-1 \\ 2 & 3 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 2 & -3 \\ y+2 & 3 \\ -z & 2 \end{bmatrix}$

$x-1=-3$ $\qquad$ $2=y+2$ $\qquad$ $-1=-z$

$\boxed{x=-2}$ $\qquad$ $\boxed{y=0}$ $\qquad$ $\boxed{z=1}$

Write the augmented matrix that represents the system:

18. $\begin{cases} x-2y+z=7 \\ 3x-5y+z=14 \\ 2x-2y-z=3 \end{cases}$

$\boxed{\begin{bmatrix} 1 & -2 & 1 & 7 \\ 3 & -5 & 1 & 14 \\ 2 & -2 & -1 & 3 \end{bmatrix}}$

Write the augmented matrices in reduced row echelon form:

19. $\begin{bmatrix} 4 & 5 & -24 \\ 2 & -3 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 2 & -3 & 0 \\ 4 & 5 & -24 \end{bmatrix}(-2) = \begin{vmatrix} 2 & -3 & 0 \\ 0 & 11 & -24 \end{vmatrix}\frac{1}{11}$

$= \begin{vmatrix} 2 & -3 & 0 \\ 0 & 1 & \frac{-24}{11} \end{vmatrix}(3) = \begin{vmatrix} 2 & 0 & -72/11 \\ 0 & 1 & -24/11 \end{vmatrix}\left(\frac{1}{2}\right) = \boxed{\begin{vmatrix} 1 & 0 & \frac{-36}{11} \\ 0 & 1 & \frac{-24}{11} \end{vmatrix}}$

20. $\begin{bmatrix} 1 & -2 & 1 & 7 \\ 0 & 1 & -2 & -7 \\ 2 & -2 & -3 & 3 \end{bmatrix}-2 = \begin{vmatrix} 1 & -2 & 1 & 7 \\ 0 & 1 & -2 & -7 \\ 0 & +2 & -5 & -11 \end{vmatrix}-2 = \begin{vmatrix} 1 & -2 & 1 & 7 \\ 0 & 1 & -2 & -7 \\ 0 & 0 & -1 & 3 \end{vmatrix}-1 =$

$= \begin{vmatrix} 1 & -2 & 1 & 7 \\ 0 & 1 & -2 & -7 \\ 0 & 0 & 1 & -3 \end{vmatrix} 2 = \begin{vmatrix} 1 & -2 & 1 & 7 \\ 0 & 1 & 0 & -13 \\ 0 & 0 & 1 & -3 \end{vmatrix} 2 \quad \begin{vmatrix} 1 & 0 & 1 & -19 \\ 0 & 1 & 0 & -13 \\ 0 & 0 & 1 & -3 \end{vmatrix}-1 = \boxed{\begin{vmatrix} 1 & 0 & 0 & -16 \\ 0 & 1 & 0 & -13 \\ 0 & 0 & 1 & -3 \end{vmatrix}}$

Write the augmented matrices in reduced row echelon form:

21. $\left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 2 & 0 & -3 & -1 \\ -1 & -1 & 2 & -1 \end{array}\right](-2)$ $= \left|\begin{array}{cccc} 1 & -1 & 1 & 0 \\ 0 & 2 & -5 & -1 \\ -1 & -1 & 2 & -1 \end{array}\right| + = \left|\begin{array}{cccc} 1 & -1 & 1 & 0 \\ 0 & 2 & -5 & -1 \\ 0 & -2 & 3 & -1 \end{array}\right| +$

$= \left|\begin{array}{cccc} 1 & -1 & 1 & 0 \\ 0 & 2 & -5 & -1 \\ 0 & 0 & -2 & -2 \end{array}\right|\left(-\frac{1}{2}\right) = \left|\begin{array}{cccc} 1 & -1 & 1 & 0 \\ 0 & 2 & -5 & -1 \\ 0 & 0 & 1 & 1 \end{array}\right| 5 = \left|\begin{array}{cccc} 1 & -1 & 1 & 0 \\ 0 & 2 & 0 & 4 \\ 0 & 0 & 1 & 1 \end{array}\right|\left(\frac{1}{2}\right) = \left|\begin{array}{cccc} 1 & -1 & 1 & 0 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 1 \end{array}\right|(-1)$

$= \left|\begin{array}{cccc} 1 & -1 & 0 & -1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 1 \end{array}\right| + = \boxed{\left|\begin{array}{cccc} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 1 \end{array}\right|}$

22. Verify that the matrices are inverses of each other:

$$A = \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}, \quad B = \begin{bmatrix} 0.8 & -0.2 \\ -0.6 & 0.4 \end{bmatrix}$$

$AB = \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix} \cdot \begin{bmatrix} 0.8 & -0.2 \\ -0.6 & 0.4 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$BA = \begin{bmatrix} 0.8 & -0.2 \\ -0.6 & 0.4 \end{bmatrix} \cdot \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Write a matrix equation for the system, and then solve using inverse matrices:

23. $\begin{cases} 3x - 2y = 0 \\ -x + y = 5 \end{cases}$ $\begin{bmatrix} 3 & -2 \\ -1 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 5 \end{bmatrix}$

$|A| = 1$

$A^{-1} = \begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix}$ $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 5 \end{bmatrix} = \boxed{\begin{bmatrix} 10 \\ 15 \end{bmatrix}}$

Write a matrix equation for the system, and then solve using inverse matrices:
(Use a Calculator to find the inverse)

24. $\begin{cases} 3x - 3y + 6z = 20 \\ x - 3y + 10z = 40 \\ -x + 3y - 5z = 30 \end{cases}$

$$\begin{bmatrix}3&-3&6\\1&-3&10\\-1&3&-5\end{bmatrix}\cdot\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}20\\40\\30\end{bmatrix}$$

$$\begin{bmatrix}x\\y\\z\end{bmatrix}=A^{-1}\cdot\begin{bmatrix}20\\40\\30\end{bmatrix}=\boxed{\begin{bmatrix}18\\118/3\\14\end{bmatrix}}$$

Write the matrix in row echelon form to solve the system:

25. $\begin{cases} x + 2y - 3z = -7 \\ 2x - 3y + z = 14 \\ 4x + y - 2z = 3 \end{cases}$

$$\left|\begin{matrix}1&2&-3&-7\\2&-3&1&14\\4&1&-2&3\end{matrix}\right|(-2)=\left|\begin{matrix}1&2&-3&-7\\0&-7&7&28\\4&1&-2&3\end{matrix}\right|(-4)$$

$$=\left|\begin{matrix}1&2&-3&-7\\0&-7&7&28\\0&-7&10&31\end{matrix}\right|(-1)=\left|\begin{matrix}1&2&-3&-7\\0&-7&7&28\\0&0&3&3\end{matrix}\right|\begin{matrix}(-\tfrac{1}{7})\\(\tfrac{1}{3})\end{matrix}=\left|\begin{matrix}1&2&-3&-7\\0&1&-1&-4\\0&0&1&1\end{matrix}\right|+$$

$$=\left|\begin{matrix}1&2&-3&-7\\0&1&0&-3\\0&0&1&1\end{matrix}\right|(-2)=\left|\begin{matrix}1&0&-3&-1\\0&1&0&-3\\0&0&1&1\end{matrix}\right|(3)=\boxed{\left|\begin{matrix}1&0&0&2\\0&1&0&-3\\0&0&1&1\end{matrix}\right|}$$

26. $\begin{cases} 2x + 3y - 12z = 1 \\ x - 2y + z = 4 \\ 4x + y - 14z = 7 \end{cases}$

$$\left|\begin{matrix}1&-2&1&4\\2&3&-12&1\\4&1&-14&7\end{matrix}\right|(-2)=\left|\begin{matrix}1&-2&1&4\\0&7&-14&-7\\4&1&-14&7\end{matrix}\right|(-4)=\left|\begin{matrix}1&-2&1&4\\0&7&-14&-7\\0&9&-18&-2\end{matrix}\right|\begin{matrix}(\tfrac{1}{7})\\(\tfrac{1}{9})\end{matrix}$$

$$=\left|\begin{matrix}1&-2&1&4\\0&1&-2&-1\\0&1&-2&-1\end{matrix}\right|(-1)=\left|\begin{matrix}1&-2&1&4\\0&1&-2&-1\\0&0&0&0\end{matrix}\right|=\boxed{\text{Infinite Solutions } (3z+2,\ 2z-1,\ z)}$$

$y - 2z = -1$

$y = 2z - 1$

$x - 2(2z-1) + z = 4$

$x - 4z + 2 + z = 4$

$x = 3z + 2$

Use cofactor and minors to find the determinant of the matrices.
Determine whether the matrix has an inverse. **(But don't find the inverse).**

27. $\begin{vmatrix}0&-1&0\\2&6&4\\1&0&3\end{vmatrix}-2 = \begin{vmatrix}0&-1&0\\0&6&-2\\1&0&3\end{vmatrix} = 1\begin{vmatrix}-1&0\\6&-2\end{vmatrix} = 1(2) = 2$

$\boxed{\text{Yes}}$

Use a calculator for #28 ☺

28. $\begin{vmatrix}1&3&3&0\\0&2&0&1\\-1&0&0&2\\1&6&4&1\end{vmatrix}$ -4 yes

Use Cramer's Rule to solve the system:

29. $\begin{cases}6x+12y = 33\\4x+7y = 20\end{cases}$

$D = \begin{vmatrix}6&12\\4&7\end{vmatrix} = 42-48 = -6$

$D_x = \begin{vmatrix}33&12\\20&7\end{vmatrix} = 231-240 = -9$ $\boxed{x = \frac{-9}{-6} = \frac{3}{2}}$

$D_y = \begin{vmatrix}6&33\\4&20\end{vmatrix} = 120-132 = -12$ $\boxed{y = \frac{-12}{-6} = 2}$ $\boxed{\left(\frac{3}{2}, 2\right)}$

30. $\begin{cases}x+2y+2z = 6\\x-2y = -1\\2x+y+3z = 7\end{cases}$

$D = \begin{vmatrix}1&2&2\\1&-2&0\\2&1&3\end{vmatrix} = 1\begin{vmatrix}-2&0\\1&3\end{vmatrix} - 2\begin{vmatrix}1&0\\2&3\end{vmatrix} + 2\begin{vmatrix}1&-2\\2&1\end{vmatrix} = 1(-6)-2(3)+2(5) = \boxed{-2}$

$D_x = \begin{vmatrix}6&2&2\\-1&-2&0\\7&1&3\end{vmatrix} = 6\begin{vmatrix}-2&0\\1&3\end{vmatrix} - 2\begin{vmatrix}-1&0\\7&3\end{vmatrix} + 2\begin{vmatrix}-1&-2\\7&1\end{vmatrix} = 6(-6)-2(-3)+2(13) = -4$ $x = \frac{-4}{-2} = \boxed{2}$

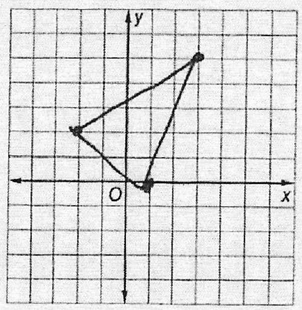
$D_y = \begin{vmatrix}1&6&2\\1&-1&0\\2&7&3\end{vmatrix} = 1\begin{vmatrix}-1&0\\7&3\end{vmatrix} - 6\begin{vmatrix}1&0\\2&3\end{vmatrix} + 2\begin{vmatrix}1&-1\\2&7\end{vmatrix} = 1(-3)-6(3)+2(9) = -3$ $y = \frac{-3}{-2} = \boxed{\frac{3}{2}}$

$D_z = \begin{vmatrix}1&2&6\\1&-2&-1\\2&1&7\end{vmatrix} = 1\begin{vmatrix}-2&-1\\1&7\end{vmatrix} - 2\begin{vmatrix}1&-1\\2&7\end{vmatrix} + 6\begin{vmatrix}1&-2\\2&1\end{vmatrix} = 1(-13)-2(9)+6(5) = -1$ $z = \frac{-1}{-2} = \boxed{\frac{1}{2}}$

$\left(2, \frac{3}{2}, \frac{1}{2}\right)$

Sketch the triangle with the given vertices and use cofactor expansion to find its area:

31- (1,0) (3,5) (−2,2)

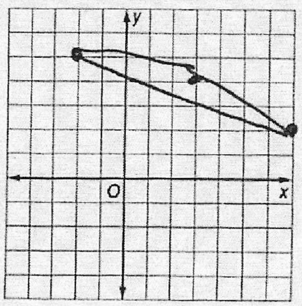


$$A = \frac{1}{2}\begin{vmatrix} 1 & 0 & 1 \\ 3 & 5 & 1 \\ -2 & 2 & 1 \end{vmatrix} = \frac{1}{2}\left(1\begin{vmatrix} 5 & 1 \\ 2 & 1 \end{vmatrix} - 0 + 1\begin{vmatrix} 3 & 5 \\ -2 & 2 \end{vmatrix}\right)$$

$$\frac{1}{2}\left((3) - 0 + (16)\right)$$

$$\boxed{= \frac{19}{2}u^2}$$

32- (−2,5) (7,2) (3, 4)



$$A = \frac{1}{2}\begin{vmatrix} -2 & 5 & 1 \\ 7 & 2 & 1 \\ 3 & 4 & 1 \end{vmatrix} = \frac{1}{2}\left(-2\begin{vmatrix} 2 & 1 \\ 4 & 1 \end{vmatrix} - 5\begin{vmatrix} 7 & 1 \\ 3 & 1 \end{vmatrix} + 1\begin{vmatrix} 7 & 2 \\ 3 & 4 \end{vmatrix}\right)$$

$$= \frac{1}{2}\left(-2(-2) - 5(4) + 1(22)\right)$$

$$\frac{1}{2}(4 - 20 + 22)$$

$$= \frac{1}{2}(6)$$

$$\boxed{= 3u^2}$$