

Your test on chapter 5 will be in two parts: a non-calculator part and a calculator part. Please complete your review without a calculator when asked.

Non-calculator Portion

Tell whether the following are examples of exponential growth or decay.

- | | | | |
|---|--|---|---|
| 1. $f(x) = 4\left(\frac{3}{8}\right)^x$
Decay | 2. $f(x) = 3\left(\frac{4}{3}\right)^x$
Growth | 3. $f(x) = 6\left(\frac{2}{3}\right)^{-x}$
Growth | 4. $f(x) = e^{-2x}$
Decay |
|---|--|---|---|

Evaluate the exact value of each of the following:

- | | | | |
|--|--|---|--|
| 5. $\log_9 3$
$9^{\frac{1}{2}} = 3$ $\frac{1}{2}$ | 6. $\log_7 7^{3x-4}$
$3x-4$ | 7. $\log_5 \frac{1}{25}$
-2 | 8. $\ln e^{-7}$
-7 |
|--|--|---|--|

Rewrite the following in logarithmic form:

- | | | | |
|--|--|--|---|
| 9. $5^{-1} = \frac{1}{5}$
$\log_5 \frac{1}{5} = -1$ | 10. $e^5 = a$
$\ln a = 5$ | 11. $\left(\frac{1}{3}\right)^{-2} = 9$
$\log_{\frac{1}{3}} 9 = -2$ | 12. $8^{\frac{2}{3}} = 4$
$\log_8 4 = \frac{2}{3}$ |
|--|--|--|---|

Rewrite the following in exponential form:

- | | | | |
|---|---|--|--|
| 13. $\log_b 5 = 3$
$b^3 = 5$ | 14. $\log_{16} 4 = \frac{1}{2}$
$16^{\frac{1}{2}} = 4$ | 15. $\ln 1 = 0$
$e^0 = 1$ | 16. $\log .01 = -2$
$10^{-2} = .01$ |
|---|---|--|--|

Write each of the following as a sum, difference and/or multiple of logarithms.

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|--|--|--|
| 17. $\log[(x-2)(x+3)^2]$
$\log(x-2) + 2\log(x+3)$ | 18. $\log_5 \left(\frac{5x^2}{25y^3}\right)$
$\log_5 5 + 2\log_5 x - \log_5 25 - 3\log_5 y$
$-1 + 2\log_5 x - 3\log_5 y$ | 19. $\ln \left(\frac{\sqrt{x-1}}{4x^3}\right)$
$\frac{1}{2}\ln(x-1) - \ln 4 - 3\ln x$ |
|--|--|--|

Write each of the following as the logarithm of a single quantity.

- | | |
|---|---|
| 20. $4\log_a 7x - 2\log_a y + \log_a 3w$
$\log_a \frac{3w(7x)^4}{y^2}$ | 21. $-3\log x + 2\log y - 3\log 4$
$\log \frac{y^2}{64x^3}$ |
| 22. $\frac{1}{2}\ln(x+1) - \frac{2}{3}\ln x - \ln 4$
$\ln \frac{\sqrt{x+1}}{4x^{2/3}}$ | 23. $6\log_b 2 + 5\log_b w - \frac{1}{2}(\log_b x + 4\log_b m)$
$\log_b \frac{64w^5}{\sqrt{x}m^4}$ |

Find the range and domain of the following functions:

24. $f(x) = -3^{x-1} - 2$

Domain: $(-\infty, \infty)$
Range: $(-\infty, -2)$

25. $f(x) = \ln x - 1$

Domain: $(0, \infty)$
Range: $(-\infty, \infty)$

26. $f(x) = 2\left(\frac{2}{3}\right)^{x+3} + 5$

Domain: $(-\infty, \infty)$
Range: $(5, \infty)$

27. $f(x) = \log_2(x+3) - 4$

Domain: $(-3, \infty)$
Range: $(-\infty, \infty)$

28. $f(x) = 4^{2x+3} - 5$

Domain: $(-\infty, \infty)$
Range: $(-5, \infty)$

29. $f(x) = -\log(3x+2) + 4$

Domain: $(-\frac{2}{3}, \infty)$
Range: $(-\infty, \infty)$

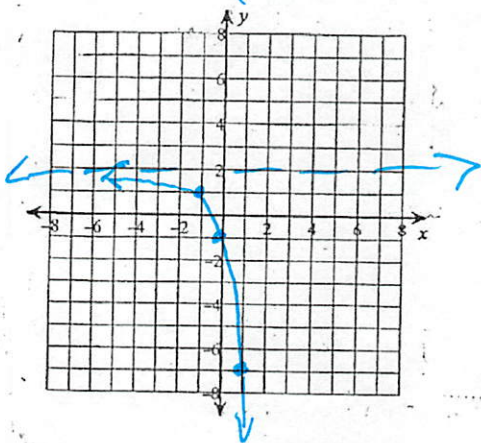
Make a table of values, carefully graph each of the following functions, and answer the accompanying questions:

30. $f(x) = -3^{x+1} + 2$

x	-1	0	1
y	1	-1	-7

$0 = -3^{x+1} + 2$
 $+3^{x+1} = 2$
 $x+1 = \log_3 2$

- A) x-intercept $(\log_3 2 - 1, 0)$
- B) y-intercept $(0, -1)$
- C) asymptote $y = 2$
- D) domain $(-\infty, \infty)$
- E) range $(-\infty, 2)$

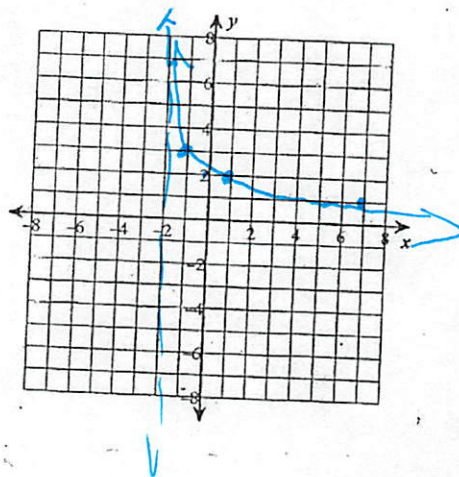


31. $f(x) = 3 - \log_3(x+2)$

x	-1	1	7
y	3	2	1

$0 = 3 - \log_3(x+2)$
 $\log_3(x+2) = 3$
 $x+2 = 3^3$

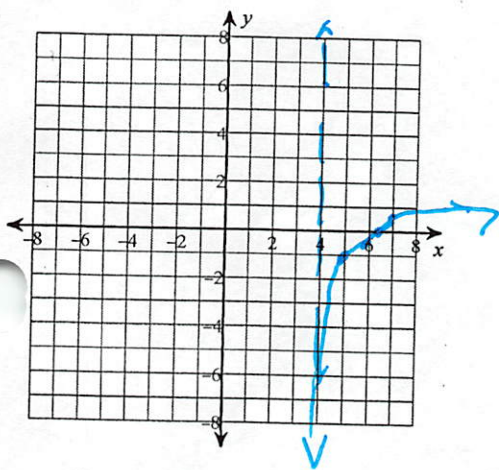
- A) x-intercept $(25, 0)$
- B) y-intercept $(0, 3 - \log_3(2))$
- C) asymptote $x = -2$
- D) domain $(-2, \infty)$
- E) range $(-\infty, \infty)$



32. $f(x) = \ln(x-4) - 1$

x	5	6	7
y	-1	-3	-9

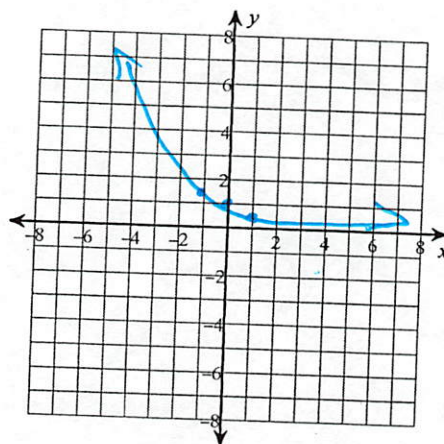
- A) x-intercept $\rightarrow 0 = \ln(x-4) - 1$
 $(4+e, 0)$
 $1 = \ln(x-4)$
 $e^1 = x-4$
- B) y-intercept \Rightarrow none $\Rightarrow \ln(0-4) - 1$
- C) asymptote $x=4$
- D) domain $(4, \infty)$
- E) range $(-\infty, \infty)$



33. $f(x) = \left(\frac{2}{3}\right)^x$

x	-1	0	1
y	3/2	1	2/3

- A) x-intercept none
- B) y-intercept $(0, 1)$
- C) asymptote $y=0$
- D) domain $(-\infty, \infty)$
- E) range $(0, \infty)$



Calculator Portion

Use the change of base formula to evaluate to the nearest thousandth.

34. $\log_5 75$

2.683

35. $\log_9 4$

0.631

36. $\log_5 9$

1.365

37. $\log_{12} 11$

0.965

Solve for x in each of the following:

38. $3e^{-5x} = 132$

$e^{-5x} = .44$
 $-5x = \ln 44$

$x \approx -0.757$

39. $e^{2x} - 6e^x + 8 = 0$

$(e^x - 2)(e^x - 4) = 0$
 $e^x = 2$ $e^x = 4$

$x \approx 0.693$
 1.386

40. $\ln x - \ln 3 = 2$

$\ln \frac{x}{3} = 2$

$\frac{x}{3} = e^2$

$x \approx 22.067$

41. $\log_3(x-2) + \log_3(x-3) = 2\log_3 x$

$\log_3(x-2)(x-3) = \log_3 x^2$

$(x-2)(x-3) = x^2$
 $x^2 - 5x + 6 = x^2$
 $-5x + 6 = 0$

$-5x = -6$
 $x = 6/5$

Use one of the following formulas to answer the question below:

$$A = P \left(1 + \frac{r}{n}\right)^{nt} \quad A = Pe^{rt} \quad T(t) = T_s + D_0 e^{-kt} \quad pH = -\log[H^+]$$

42. Find the interest rate on an account that doubles your money in 6 years when the interest is compounded continuously.

$$A = Pe^{rt}$$

$$2 = e^{6r}$$

$$\ln 2 = 6r$$

$$r \approx .1155$$

$$\boxed{11.55\%}$$

43. The half-life of an element is 30 years. How many years will it take until only 2% remains?

$$\frac{1}{2} = e^{30r}$$

$$\ln \frac{1}{2} = 30r$$

$$r \approx -2.31\%$$

$$\rightarrow .02 = e^{-.0231t}$$

$$\ln .02 = -.0231t$$

$$\boxed{169.35 \text{ years}}$$

44. The pH of carbonated water is 3.9 and the pH of household ammonia is 11.9. What are their hydrogen-ion concentrations?

$$3.9 = -\log(H^+)$$

$$-3.9 = \log(H^+)$$

$$10^{-3.9} \approx \boxed{1.259 \times 10^{-4}}$$

$$11.9 = -\log(H^+)$$

$$-11.9 = \log(H^+)$$

$$10^{-11.9} \approx \boxed{1.259 \times 10^{-12}}$$

45. If you deposit \$500 in a savings account paying 3.2% interest, compounded monthly, how much money would you have at the end of 5 years?

$$A = 500 \left(1 + \frac{.032}{12}\right)^{12(5)}$$

$$\boxed{\$586.63}$$

46. Find the number of years it takes to have 10 times your initial investment saved at 10% interest compounded annually.

$$10 = \left(1 + \frac{.1}{1}\right)^t$$

$$10 = (1.1)^t$$

$$\log_{1.1} 10 = t$$

$$\boxed{24.16 \text{ years}}$$

47. The population of a specific animal is decreasing. Ten years ago there were 1000 counted in a specific area. Today there are only 50. How many years will it take until only 1 remains?

$$50 = 1000 e^{10r}$$

$$.05 = e^{10r}$$

$$\ln(.05) = r$$

$$r \approx -.300$$

$$1 = 50 e^{-.3t}$$

$$\ln(.02) = -.3t$$

$$t \approx 13 \text{ years}$$

$$\boxed{23 \text{ total years}}$$

48. A hard-boiled egg at temperature of 96° C is placed in 16° C water to cool. Four minutes later the temperature of the egg is 45° C. When will the temperature of the egg be 20° C?

$$45 = 16 + 80 e^{-k(4)}$$

$$29 = 80 e^{-4k}$$

$$\ln(.3625) = -4k$$

$$k \approx 0.254$$

$$20 = 16 + 80 e^{-.254(t)}$$

$$4 = 80 e^{-.254t}$$

$$\ln(.05) = -.254t$$

$$\boxed{t \approx 11.8 \text{ minutes}}$$