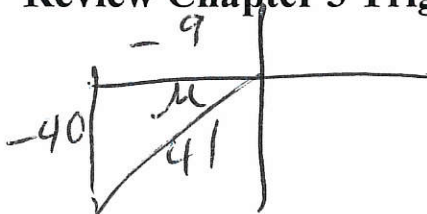


Name _____

Review Chapter 3 Trigonometry

1. $\sec u = -\frac{41}{9}, \pi < u < \frac{3\pi}{2}$

Find each of the following:



a) $\tan u$

Answer a) $\frac{40}{9}$

b) $\sin 2u$ $2 \sin u \cos u$

Answer b) $\frac{720}{1681}$

c) $\cos 2u$ $\cos^2 u - \sin^2 u$

Answer c) $\frac{-1519}{1681}$

d) $\cos \frac{u}{2}$ $-\sqrt{\frac{1 + \cos u}{2}}$

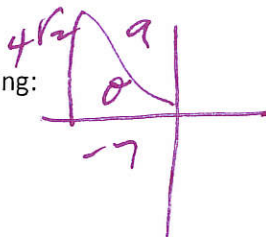
Answer d) $-\frac{4\sqrt{41}}{41}$

e) $\tan^2 u$

$\left(\frac{40}{9}\right)^2$

Answer e) $\frac{1600}{81}$

2. $\cos \theta = -\frac{7}{9}, \tan \theta < 0$ Find each of the following:



a) $\sin \theta$

Answer a) $\frac{4\sqrt{2}}{9}$

b) $\sin 2\theta$ $2 \sin \theta \cos \theta$

Answer b) $\frac{-56\sqrt{2}}{81}$

c) $\cos 2\theta$ $\cos^2 \theta - \sin^2 \theta$

Answer c) $\frac{17}{81}$

d) $\tan^2 \theta$ $\left(\frac{4\sqrt{2}}{-7}\right)^2$

Answer d) $\frac{32}{49}$

e) $\tan\left(\frac{\theta}{2}\right)$

$\frac{\sin \theta}{1 + \cos \theta}$

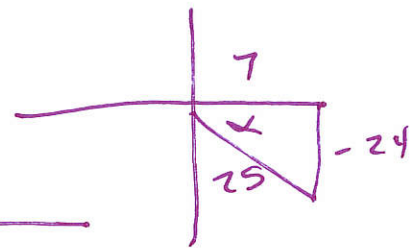
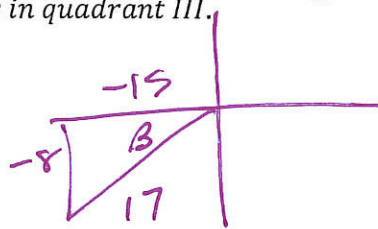
Answer e) $2\sqrt{2}$

For problems 3 and 4: $\cos \alpha = \frac{7}{25}$, α is in quadrant IV.

$\sin \beta = -\frac{8}{17}$, β is in quadrant III.

3. Find $\cos(\alpha - \beta)$

$$\cos \alpha \cos \beta + \sin \alpha \sin \beta$$



Answer 87/425

4. Find $\tan(2\beta)$

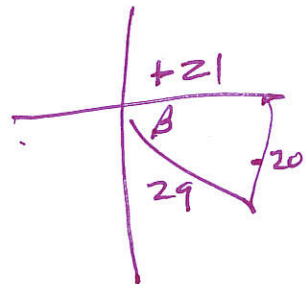
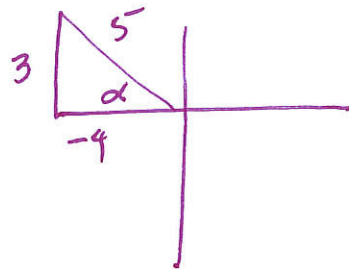
$$\frac{2 \tan \beta}{1 - \tan^2 \beta}$$

Answer 240/161

For problems 5 and 6:

$\cos \alpha = -\frac{4}{5}$, α is in quadrant II.

$\sin \beta = \frac{20}{29}$, β is in quadrant IV.



5. Find $\sin(\alpha + \beta)$

$$\sin \alpha \cos \beta + \sin \beta \cos \alpha$$

$$\frac{3}{5} \left(\frac{21}{29} \right) + \left(\frac{20}{29} \right) \left(-\frac{4}{5} \right)$$

$$\frac{63}{145} + \frac{80}{145} =$$

Answer 143/145

6. Find $\tan(\alpha - \beta)$

$$\frac{\tan \alpha - \tan \beta}{1 + \tan \alpha \tan \beta} = \frac{-\frac{3}{4} - \left(\frac{20}{21} \right)}{1 + \left(-\frac{3}{4} \right) \left(\frac{20}{21} \right)}$$

$$= \frac{-\frac{3}{4} - \frac{20}{21}}{1 - \frac{60}{84}} \cdot \left(\frac{84}{84} \right) = \frac{-63 + 80}{84 + 60} = \frac{17}{144}$$

Answer 17/144

7. Find $\cos 75^\circ$ in exact form. Hint: $\cos 75^\circ = \cos(135^\circ - 60^\circ)$ or $\cos 75^\circ = \cos(30^\circ + 45^\circ)$

$$\cos 30^\circ \cdot \cos 45^\circ - \sin 30^\circ \sin 45^\circ$$

$$\frac{\sqrt{3}}{2} \left(\frac{\sqrt{2}}{2} \right) - \left(\frac{1}{2} \right) \left(\frac{\sqrt{2}}{2} \right)$$

Answer $\frac{\sqrt{6} - \sqrt{2}}{4}$

8. Find $\sin \frac{13\pi}{12}$ in exact form. Hint: $\sin \frac{13\pi}{12} = \sin \left(\frac{5\pi}{6} + \frac{\pi}{4} \right)$

$$\sin \frac{5\pi}{6} \cos \frac{\pi}{4} + \sin \frac{\pi}{4} \cos \frac{5\pi}{6}$$

$$\frac{1}{2} \left(\frac{\sqrt{2}}{2} \right) + \frac{\sqrt{2}}{2} \left(-\frac{\sqrt{3}}{2} \right)$$

Answer $\frac{\sqrt{2} - \sqrt{6}}{4}$

9. Find $\sin \frac{7\pi}{12}$ in exact form. Hint: $\sin \frac{7\pi}{12} = \sin \frac{7\pi}{12}$

$$\sin \frac{7\pi}{12} = + \sqrt{\frac{1 - \left(-\frac{\sqrt{3}}{2}\right)}{2}} = \sqrt{\frac{2 + \sqrt{3}}{4}}$$

$$= + \sqrt{\frac{1 - \cos \frac{7\pi}{6}}{2}}$$

Answer $\frac{\sqrt{2 + \sqrt{3}}}{2}$

10. Find $\sin \frac{5\pi}{8}$ in exact form. Hint: $\sin \frac{5\pi}{8} = \sin \frac{5\pi}{8}$

$$\sin \frac{5\pi}{8} = + \sqrt{\frac{1 - \cos \frac{5\pi}{4}}{2}} = \sqrt{\frac{1 - \left(-\frac{\sqrt{2}}{2}\right)}{2}} \cdot \frac{2}{2}$$

$$= \sqrt{\frac{2 + \sqrt{2}}{4}}$$

Answer $\frac{\sqrt{2 + \sqrt{2}}}{2}$

Find all of the solutions in the interval $0 \leq x < 2\pi$ For each of the following equations

11. $2\sec^2 x + \tan^2 x - 3 = 0$

$$2(1 + \tan^2 x) + \tan^2 x - 3 = 0$$

$$2 + 2\tan^2 x + \tan^2 x - 3 = 0$$

$$3\tan^2 x - 1 = 0 \quad \tan x = \pm \frac{\sqrt{3}}{3}$$

$$\tan^2 x = \frac{1}{3}$$

Answer $\frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

12. $3\cot^2 x \sec^2 x - \sec^2 x - 6\cot^2 x = 0$

$$\sec^2 x (3\cot^2 x - 1) - 2(3\cot^2 x + 1) = 0$$

$$(3\cot^2 x - 1)(\sec^2 x - 2) = 0$$

$$\cot x = \pm \frac{\sqrt{3}}{3} \quad \sec^2 x = 2$$

$$\cos x = \pm \frac{\sqrt{2}}{2}$$

Answer $\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$

Answer $\frac{\pi}{4}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{4}$

13. $\sin 2x \sin x - \cos x = 0$

$$\downarrow$$

$$2 \sin x \cos x \cdot \sin x - \cos x = 0$$

$$\cos x (2 \sin^2 x - 1) = 0$$

$$\cos x = 0 \quad \sin x = \pm \frac{\sqrt{2}}{2}$$

$$\frac{\pi}{2}, \frac{3\pi}{2}$$

Answer $\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

14. $\cos\left(x + \frac{\pi}{4}\right) + \cos\left(x - \frac{\pi}{4}\right) = 1$

$$\cos x \cos \frac{\pi}{4} - \sin x \sin \frac{\pi}{4}$$

$$\cos x \cos \frac{\pi}{4} + \sin x \sin \frac{\pi}{4}$$

$$\cos x = \frac{\sqrt{2}}{2}$$

$$\frac{2 \cos x \cdot \left(\frac{\sqrt{2}}{2}\right)}{2 \cos x \cdot \left(\frac{\sqrt{2}}{2}\right)} = 1 \quad \sqrt{2} \cos x = 1$$

Answer $\frac{\pi}{4}, \frac{7\pi}{4}$

15. $\csc^2 x - 2 = 0$

$$\csc^2 x = 2$$

$$\csc x = \pm \sqrt{2}$$

$$\sin x = \pm \frac{\sqrt{2}}{2}$$

Answer $\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

16. $(3 \tan^2 x - 1)(\tan^2 x - 3) = 0$

$$3 \tan^2 x - 1 = 0 \quad \tan^2 x - 3 = 0$$

$$\tan x = \pm \frac{\sqrt{3}}{3} \quad \tan x = \pm \sqrt{3}$$

Answer $\frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

17. $2 \sin^2 x = 2 + \cos x$

$$2(1 - \cos^2 x) = 2 + \cos x$$

$$\cos x (2 \cos x + 1) = 0$$

$$2 - 2 \cos^2 x = 2 + \cos x$$

$$\cos x = 0 \quad \cos x = -\frac{1}{2}$$

$$0 = 2 \cos^2 x + \cos x$$

Answer $\frac{\pi}{2}, \frac{3\pi}{2}, \frac{2\pi}{3}, \frac{4\pi}{3}$

18. $2 \sin x + \sqrt{3} = 0$

$$2 \sin x = -\sqrt{3}$$

$$\sin x = -\frac{\sqrt{3}}{2}$$

Answer $\frac{4\pi}{3}, \frac{5\pi}{3}$

Verify each of the following:

19. $\frac{\cos x \csc x}{\cot^2 x} = \tan x$

$$\frac{\cos x \cdot \frac{1}{\sin x}}{\frac{\cos^2 x}{\sin^2 x}} = \frac{\cos}{\sin x} \cdot \frac{\sin^2 x}{\cos^2 x} = \frac{\sin x}{\cos x} = \tan x$$

20. $\frac{\csc x}{\sin x} - \frac{\cot x}{\tan x} = 1$

$$\frac{\frac{1}{\sin x}}{\sin x} - \frac{\frac{\cos x}{\sin x}}{\frac{\sin x}{\cos x}} = \frac{1}{\sin^2 x} - \frac{\cos^2 x}{\sin^2 x} = \frac{1 - \cos^2 x}{\sin^2 x} = \frac{\sin^2 x}{\sin^2 x} = 1$$

21. $\frac{1}{\sin x} - \sin x = \frac{\cos^2 x}{\sin x}$

$$\frac{1}{\sin x} - \frac{\sin^2 x}{\sin x} = \frac{1 - \cancel{\cos^2 x}}{\sin x} = \frac{\cos^2 x}{\sin x}$$

22. $\sec^2 y - \cot^2\left(\frac{\pi}{2} - y\right) = 1$

$$\sec^2 y - \tan^2 y = 1$$

23. $\tan\left(\frac{\pi}{4} - \theta\right) = \frac{1 - \tan \theta}{1 + \tan \theta}$

$$\frac{\tan \frac{\pi}{4} - \tan \theta}{1 + \tan \frac{\pi}{4} \tan \theta} = \frac{1 - \tan \theta}{1 + \tan \theta}$$

24. $\frac{\sec x + \tan x}{\sec x - \tan x} = (\sec x + \tan x)^2$

$$\frac{(\sec x + \tan x)}{(\sec x - \tan x)} \cdot \frac{(\sec x + \tan x)}{(\sec x + \tan x)} = \frac{(\sec x + \tan x)^2}{\sec^2 x - \tan^2 x} = \frac{(\sec x + \tan x)^2}{1}$$

25. $\frac{\cos \theta \cot \theta}{1 - \sin \theta} - 1 = \csc \theta$

$$\frac{\cos \theta \cdot \frac{\cos \theta}{\sin \theta}}{1 - \sin \theta} - 1 = \frac{\cos^2 \theta - \sin \theta + \sin^2 \theta}{\sin \theta - \sin^2 \theta} = \frac{1 - \sin \theta}{\sin \theta (1 - \sin \theta)} = \frac{1}{\sin \theta} = \csc \theta$$

$$\frac{\cos^2 \theta}{\sin \theta} - 1 + \sin \theta = \frac{\cos^2 \theta}{\sin \theta} - 1 + \sin \theta = \frac{\cos^2 \theta - \sin \theta + \sin^2 \theta}{\sin \theta} = \frac{1 - \sin \theta}{\sin \theta}$$

$\cos \theta : 0 \leq \theta \leq \pi$

Use: $\sin \theta : -\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$

$\tan \theta : -\frac{\pi}{2} < \theta < \frac{\pi}{2}$

Do not use a calculator on this part of the test

26. Find the simplified exact value of each of the following expressions:

a. $\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$

Answer a) $3\pi/6$

b. $\tan^{-1}0$

Answer b) 0

c. $\csc^{-1}(-1)$

Answer c) $-\frac{\pi}{2}$

d. $\sin^{-1}\left(\frac{\sqrt{2}}{2}\right)$

Answer d) $\frac{\pi}{4}$

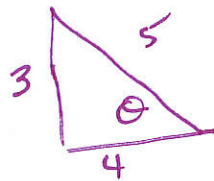
e. $\sec^{-1}\sqrt{2}$

Answer e) $+\frac{\pi}{4}$

f. $\cot^{-1}(-1)$

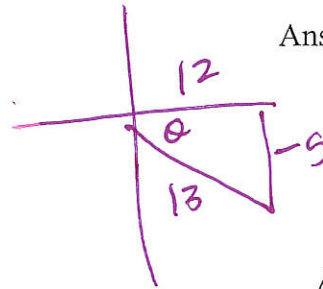
Answer f) $-\frac{\pi}{4}$

g. $\cos\left(\cos^{-1}\left(\frac{4}{5}\right)\right)$



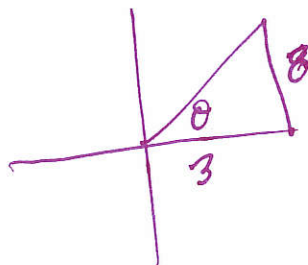
Answer g) $\frac{4}{5}$

h. $\cos\left(\sin^{-1}\left(-\frac{5}{13}\right)\right)$



Answer h) $\frac{12}{13}$

i. $\tan\left(\cot^{-1}\left(\frac{3}{8}\right)\right)$



Answer i) $\frac{8}{3}$