

## Chapter 9/10 Review

Find a formula for the  $n$ th term of an arithmetic sequence with

1)  $a_1 = 7$  and  $d = -3$

Find the formula for the  $n$ th term of an arithmetic sequence, then find the 35th term.

2) The first 3 terms are 5, 9, and 13.

Find the  $n$ th term of an arithmetic sequence with

3)  $a_4 = 26$  and  $a_8 = 50$ .

Find the sum of the first 12 terms of an arithmetic series with

4)  $a_1 = 4$ ,  $d = 7$

**Determine the number of terms  $n$  in each arithmetic series.**

5)  $a_1 = -2$ ,  $a_n = 8$ ,  $S_n = 18$

6)  $a_1 = 25$ ,  $d = 10$ ,  $S_n = 385$

7)  $\sum_{m=1}^n (2m + 1) = 120$

**Find the sum of the first 20 terms of the arithmetic series.**

8)  $18 + 23 + 28 + 33 \dots$

**Evaluate the arithmetic series described.**

9)  $\sum_{m=1}^{45} (8m - 15)$

**Find the 8th term of the geometric sequence**

10) with  $a_1 = 5$  and  $r = 3$

**Find the 11th term of the geometric sequence with**

11)  $a_1 = -81$  and  $r = \frac{1}{3}$

**Find a formula for the nth term of a geometric sequence when**

12) the first 3 terms are  $\frac{1}{4}, 1, 4, \dots$ . Then find the 11th term.

**Find a formula for the nth term of a geometric sequence if**

13)  $a_3 = 18$  and  $a_6 = 486$

**Evaluate each geometric series described.**

14)  $a_1 = 2, r = 5, n = 9$

15)  $1 + \frac{2}{3} + \frac{4}{9} \dots, n = 5$

16)  $4 + 16 + 64 + 256 \dots, n = 7$

**Determine the number of terms  $n$  in each geometric series.**

17)  $1 - 3 + 9 - 27 \dots, S_n = -182$

18)  $\sum_{m=1}^n 135 \cdot \left(\frac{1}{3}\right)^{m-1} = 200$

**Evaluate each infinite geometric series described.**

19)  $64 + 16 + 4 + 1 \dots$

20)  $-2 + \frac{4}{3} - \frac{8}{9} + \frac{16}{27} \dots$

**Write each of the following in summation notation:**

21)  $-8 - 3 + 2 + \dots + 92$

22)  $4 - 8 + 16 - 32 + \dots - 2048$

**Expand each of the following binomials using Pascal's Triangle or the Binomial Theorem.**

23)  $(4x + 1)^4$

24)  $(5x - 4y)^3$

25)  $(x + 3y^2)^3$

26)  $(2u + v)^7$

**Find the 4th term of the binomial expansion**

27)  $(2x + 3y)^9$

**Find the 27th term of the binomial expansion**

28)  $(2x - y)^{30}$

**Find the term with  $a^3$  in the binomial expansion**

29)  $(4a - 2b)^{14}$

**Factor the polynomial using the binomial theorem**

30)  $32x^5 + 80x^4y + 80x^3y^2 + 40x^2y^3 + 10xy^4 + y^5$

**Find the probability of each event.**

- 32) A technician is launching fireworks near the end of a show. Of the remaining fourteen fireworks, nine are blue and five are red. If she launches nine of them in a random order, what is the probability that all of them are blue?
- 33) A chemistry lab requires students to identify chemical compounds by using various tests. Each student is given samples of three different compounds, labeled A, B, and C. Each student is also given a list of twelve possible compounds. If a student does not perform the tests and randomly chooses three from the list, what is the probability that she guesses all three correctly?

**Determine if the scenario involves mutually exclusive events. Then find the probability.**

- 34) A jar contains three blue marbles numbered one to three. The jar also contains five red marbles numbered one to five. You randomly pick a marble. It is blue or has an even number.
- 35) A cooler contains eleven bottles of sports drink: three lemon-lime flavored, four orange flavored, and four fruit-punch flavored. You randomly grab a bottle. It is a lemon-lime or an orange.

**Find the probability of each event.**

- 36) You've purchased a lottery ticket and your numbers are: 1-6-2. A lottery official randomly selects three balls from a set of seven balls that are numbered from #1 to #7. To win, your numbers must match the selected numbers in order. What is the probability of winning the lottery?
- 37) A bag contains six real diamonds and two fake diamonds. If six diamonds are picked from the bag at random, what is the probability that all of them are real?
- 38) A nature preserve has a population of fifteen black bears. They have been tagged #1 through #15, so they can be observed over time. Two of them are randomly selected and captured for evaluation. What is the probability that bears #3 and #5 are captured for evaluation?

## Chapter 9 Review

Find a formula for the  $n$ th term of an arithmetic sequence with

1)  $a_1 = 7$  and  $d = -3$

$$a_n = 7 + (n-1)(-3)$$

Find the formula for the  $n$ th term of an arithmetic sequence, then find the 35th term.

2) The first 3 terms are 5, 9, and 13.

$a = 5$

$r = 4$

$$a_n = 5 + (n-1)4$$

$$a_{35} = 5 + 34(4) = 141$$

Find the  $n$ th term of an arithmetic sequence with

3)  $a_4 = 26$  and  $a_8 = 50$ .

$$d = \frac{50 - 26}{8 - 4} = \frac{24}{4} = 6$$

$$a_n = 8 + (n-1)6$$

$$a_1 = 26 - 3(6) = 8$$

Find the sum of the first 12 terms of an arithmetic series with

4)  $a_1 = 4$ ,  $d = 7$

$$S_n = \frac{n}{2}(a_1 + a_n)$$

$$a_{12} = 4 + (11)7$$

$$= 81$$

$$S_n = \frac{12}{2}(4 + 81) = \boxed{510}$$

Determine the number of terms  $n$  in each arithmetic series.

5)  $a_1 = -2$ ,  $a_n = 8$ ,  $S_n = 18$

$$S_n = \frac{n}{2}(a_1 + a_n)$$

$$18 = \frac{n}{2}(-2 + 8)$$

$$36 = 6n$$

$$\boxed{n = 6}$$

7)  $\sum_{m=1}^n (2m+1) = 120$

$$S_n = \frac{n}{2}(2a + (n-1)d)$$

$$a_1 = 2(1) + 1 = 3$$

$$120 = \frac{n}{2}(6 + (n-1)2)$$

$$d = 2$$

$$240 = n(2n + 4)$$

$$2n^2 + 4n - 240 = 0$$

$$n^2 + 2n - 120 = 0$$

$$(n+12)(n-10) = 0$$

$$S_n = 120$$

$$\boxed{n = 10}$$

6)  $a_1 = 25$ ,  $d = 10$ ,  $S_n = 385$

$$S_n = \frac{n}{2}(2a + (n-1)d)$$

$$385 = \frac{n}{2}(50 + (n-1)10)$$

$$770 = n(10n + 40)$$

$$10n^2 + 40n - 770 = 0$$

$$n^2 + 4n - 77 = 0$$

$$(n+11)(n-7) = 0$$

$$\boxed{n = 7}$$

Find the sum of the first 20 terms of the arithmetic series.

$$8) 18 + 23 + 28 + 33, \dots \quad S_n = \frac{n}{2} (2a + (n-1)d)$$

$$S_n = \frac{20}{2} (36 + (19)5)$$

$$= 1310$$

Evaluate the arithmetic series described.

$$9) \sum_{m=1}^{45} (8m - 15) \quad S_n = \frac{45}{2} (a_1 + a_{45})$$

$$a_1 = -7 \quad = \frac{45}{2} (-7 + 345)$$

$$a_{45} = 8(45) - 15 = 345 \quad = 7605$$

Find the 8th term of the geometric sequence

$$10) \text{ with } a_1 = 5 \text{ and } r = 3 \quad a_n = a(r)^{n-1}$$

$$a_8 = 5(3)^7$$

$$= 10935$$

Find the 11th term of the geometric sequence with

$$11) a_1 = -81 \text{ and } r = \frac{1}{3} \quad a_{11} = -81 \left(\frac{1}{3}\right)^{10} = \frac{-81}{59049} = -\frac{1}{729}$$

Find a formula for the nth term of a geometric sequence when

12) the first 3 terms are  $\frac{1}{4}, 1, 4, \dots$ . Then find the 11th term.

$$a = \frac{1}{4}$$

$$r = 4$$

$$a_n = \frac{1}{4}(4)^{n-1} \quad a_{11} = \frac{1}{4}(4)^{10} = \frac{4^{10}}{4} = 4^9 = 262,144$$

Find a formula for the nth term of a geometric sequence if

$$13) a_3 = 18 \text{ and } a_6 = 486 \quad a_1 = \frac{18}{3^2} = 2$$

$$r^3 = \frac{486}{18} = 27$$

$$r = 3$$

$$a_n = 2(3)^{n-1}$$



Evaluate each geometric series described.

14)  $a_1 = 2, r = 5, n = 9$

$$S_n = a \left( \frac{1-r^n}{1-r} \right)$$

$$2 \left( \frac{1-5^9}{1-5} \right) = \boxed{1976,562}$$

15)  $1 + \frac{2}{3} + \frac{4}{9} \dots, n = 5$

$$1 \left( \frac{1 - \left(\frac{2}{3}\right)^5}{1 - \frac{2}{3}} \right) = \boxed{\frac{211}{81}}$$

16)  $4 + 16 + 64 + 256 \dots, n = 7$

$n = 7$   
 $r = 4$   
 $a = 4$

$$4 \left( \frac{1-4^7}{1-4} \right) = \boxed{21,844}$$

Determine the number of terms  $n$  in each geometric series.

17)  $1 - 3 + 9 - 27 \dots, S_n = -182$

$$1 \left( \frac{1 - (-3)^n}{1 - (-3)} \right) = -182$$

$$1 - (-3)^n = -728 \quad \boxed{n = 6}$$

$$(-3)^n = 729$$

Evaluate each infinite geometric series described.

19)  $64 + 16 + 4 + 1 \dots$

$$S_n = \frac{a}{1-r}$$

$r = \frac{1}{4}$

$$S_n = \frac{64}{1 - \frac{1}{4}} = \boxed{85 \frac{1}{3}}$$

or  $\frac{256}{3}$

18)  $\sum_{m=1}^n 135 \cdot \left(\frac{1}{3}\right)^{m-1} = 200$

$$200 = 135 \left( \frac{1 - \left(\frac{1}{3}\right)^n}{1 - \frac{1}{3}} \right)$$

$$\frac{40}{27} = \frac{1 - \left(\frac{1}{3}\right)^n}{\frac{2}{3}}$$

$$\frac{80}{81} = 1 - \left(\frac{1}{3}\right)^n$$

$$\frac{1}{81} = \left(\frac{1}{3}\right)^n \quad \boxed{n = 4}$$

20)  $-2 + \frac{4}{3} - \frac{8}{9} + \frac{16}{27} \dots$

$r = -\frac{2}{3}$

$$\frac{-2}{1 - -\frac{2}{3}} = \frac{-2}{\frac{5}{3}}$$

$$\boxed{\frac{-6}{5}}$$

Write each of the following in summation notation:

21)  $-8 - 3 + 2 + \dots + 92$

$d = 5$   
 $a_n = a + (n-1)d$   
 $a = -8$   
 $92 = -8 + (n-1)5$   
 $-8 + 5n - 5$

$105 = 5n$   
 $n = 21$

$$\sum_{n=1}^{21} -8 + (n-1)5$$

22)  $4 - 8 + 16 - 32 + \dots - 2048$

$r = -2$   
 $a = 4$

$a_n = a(r)^{n-1}$   
 $-2048 = 4(-2)^{n-1}$   
 $-512 = (-2)^{n-1}$   
 $9 = n-1 \quad n = 10$

$$\sum_{n=1}^{10} 4(-2)^{n-1}$$

Expand each of the following binomials using Pascal's Triangle or the Binomial Theorem.

23)  $(4x + 1)^4$

1 4 6 4 1

$(4x)^4 (4x)^3 (4x)^2 (4x)$

$$\boxed{256x^4 + 256x^3 + 96x^2 + 16x + 1}$$

24)  $(5x - 4y)^3$

1 3 3 1

$(5x)^3 (5x)^2 (5x)$   
 $(-4y) (-4y)^2 (-4y)^3$

$$\boxed{-125x^3 - 300x^2y + 240xy^2 - 64y^3}$$

25)  $(x + 3y^2)^3$

$$\begin{matrix} 1 & 3 & 3 & 1 \\ x^3 & x^2 & x & \\ & 3y^2 & (3y^2)^2 & (3y^2)^3 \end{matrix}$$

$$x^3 + 9x^2y^2 + 27xy^4 + 27y^6$$

26)  $(2u + v)^7$

$$\begin{matrix} 1 & 7 & 21 & 35 & 35 & 21 & 7 & 1 \\ (2u)^7 & (2u)^6 & (2u)^5 & (2u)^4 & (2u)^3 & (2u)^2 & (2u) & \\ & v^1 & v^2 & v^3 & v^4 & v^5 & v^6 & v^7 \end{matrix}$$

$$128u^7 + 448u^6v + 672u^5v^2 + 560u^4v^3 + 280u^3v^4 + 84u^2v^5 + 14uv^6 + v^7$$

Find the 4th term of the binomial expansion

27)  $(2x + 3y)^9$       9      9      9       $\binom{9}{3} = 84$

$$\begin{matrix} (2x)^6 \\ (3y)^3 \end{matrix} = 145152x^6y^3$$

Find the 27th term of the binomial expansion

28)  $(2x - y)^{30}$        $\binom{30}{26} = 27405$

$$\begin{matrix} (2x)^4 \\ (-y)^{26} \end{matrix} = 438,480x^4y^{26}$$

Find the term with  $a^3$  in the binomial expansion

29)  $(4a - 2b)^{14}$        $\binom{14}{11} = 364$

$$\begin{matrix} (4a)^3 \\ (-2b)^{11} \end{matrix} = -47,710,208a^3b^{11}$$

Factor the polynomial using the binomial theorem

30)  $32x^5 + 80x^4y + 80x^3y^2 + 40x^2y^3 + 10xy^4 + y^5$

$$(2x + y)^5$$

**Find the probability of each event.**

- 32) A technician is launching fireworks near the end of a show. Of the remaining fourteen fireworks, nine are blue and five are red. If she launches nine of them in a random order, what is the probability that all of them are blue?

$$\frac{1}{{}_{14}C_9} = \frac{1}{2002} = .05\%$$

- 33) A chemistry lab requires students to identify chemical compounds by using various tests. Each student is given samples of three different compounds, labeled A, B, and C. Each student is also given a list of twelve possible compounds. If a student does not perform the tests and randomly chooses three from the list, what is the probability that she guesses all three correctly?

$$\frac{1}{{}_{12}P_3} = \frac{1}{1320} = .076\%$$

**Determine if the scenario involves mutually exclusive events. Then find the probability.**

- 34) A jar contains three blue marbles numbered one to three. The jar also contains five red marbles numbered one to five. You randomly pick a marble. It is blue or has an even number.

$$\frac{3}{8} + \frac{3}{8} - \frac{1}{8} = \frac{5}{8} = 62.5\%$$

- 35) A cooler contains eleven bottles of sports drink: three lemon-lime flavored, four orange flavored, and four fruit-punch flavored. You randomly grab a bottle. It is a lemon-lime or an orange.

$$\frac{3}{11} + \frac{4}{11} = \frac{7}{11} \approx 63.6\%$$

**Find the probability of each event.**

- 36) You've purchased a lottery ticket and your numbers are: 1-6-2. A lottery official randomly selects three balls from a set of seven balls that are numbered from #1 to #7. To win, your numbers must match the selected numbers in order. What is the probability of winning the lottery?

$$\frac{1}{{}_{7}P_3} = \frac{1}{210} \approx 47.6\%$$

- 37) A bag contains six real diamonds and two fake diamonds. If six diamonds are picked from the bag at random, what is the probability that all of them are real?

$$\frac{1}{{}_{8}C_6} = \frac{1}{28} = 3.57\%$$

- 38) A nature preserve has a population of fifteen black bears. They have been tagged #1 through #15, so they can be observed over time. Two of them are randomly selected and captured for evaluation. What is the probability that bears #3 and #5 are captured for evaluation?

$$\frac{1}{{}_{15}C_2} = \frac{1}{105} \approx .95\%$$

State if each scenario involves a permutation or a combination. Then find the number of possibilities.

31) A group of 25 people are going to run a race. The top three runners earn gold, silver, and bronze medals.

$$P \quad {}_{25}P_3 = 13,800$$

32) A group of 35 people are going to run a race. The top 11 finishers advance to the finals.

$$C \quad {}_{35}C_{11} = 417,225,900$$

Determine if the scenario involves mutually exclusive events. Then find the probability.

33) A box of chocolates contains five milk chocolates and seven dark chocolates. Two of the milk chocolates and five of the dark chocolates have peanuts inside. You randomly select and eat a chocolate. It is a milk chocolate or has no peanuts inside.

not mutually exclusive

$$\frac{5}{12} + \frac{5}{12} - \frac{3}{12} = \frac{7}{12} \approx 58.3\%$$

34) A litter of kittens consists of three gray kittens, two black kittens, and two mixed-color kittens. You randomly pick one kitten. The kitten is gray or mixed-color.

mutually exclusive

$$\frac{3}{7} + \frac{2}{7} = \frac{5}{7} \approx 71.4\%$$

Find the probability of each event.

35) Perry and Daniel each purchase one raffle ticket. If a total of twelve raffle tickets are sold, what is the probability that Perry wins the grand prize and Daniel wins the second prize?

one way for them to be 1st and 2nd

possible perm.  $\frac{1}{{}_{12}P_2} = \frac{1}{132} \approx 0.76\%$

36) A bag contains eight real diamonds and five fake diamonds. If six diamonds are picked from the bag at random, what is the probability that exactly four of them are real?

$$\frac{{}_8C_4 \cdot {}_5C_2}{{}_{13}C_6} = \frac{700}{1716} \approx 40.79\%$$

37) A basketball player has a 50% chance of making each free throw. What is the probability that the player makes exactly four out of ten free throws?

Possible combinations

50% chance w/ 10 shots

$$\frac{{}_{10}C_4}{2^{10}} \approx 20.51\%$$