Section 6.2
I can identify appropriate methods to solve logarithmic equations.

Methods:
By converting to exponential form
By condensing an expression
By taking a log of both sides
By changing the bases

Properties of Logarithms:

$a^x = a^y$ *if only if* $x =$ y

$\log_b x = \log_b y$ *if and only if* $x = y$

Example:
If $\log_3 x = \log_3 8$ then x = 8

$\log_7(6x-16) = \log_7(x-1)$

$6x-16 = x-1$
$5x = 15$
$x = 3$

(Check for extraneous solutions!)

$\log_9(x^2-4x) = \log_9(3x-10)$

$x^2-4x = 3x-10$
$x^2-7x+10 = 0$
$(x-5)(x-2)=0$
$x=2, x=5$

Check
$(2)^2-4(2) = 0$ ✓
not a solution

$5^2-4(5) = 5$ ✓
$3(5)-10=5$ ✓

You try ☺
$\log_3(x-5) = \log_3(3x-25)$

$\log_7(x^2-4) = \log_7(-x+2)$

Since $a^0 = 1$ & $a^1 = a$

$\log_a 1 = 0$ $\log_a a = 1$

$\log_b x = y$ *and* $b^y = x$

Substitute $x = b^y$
$\log_b b^y = y$ Or Substitute $\log_b x = y$
$b^{\log_b x} = x$

Solve: Exponentiate each side.

$\log_8(x-5) = \frac{2}{3}$

$8^{\log_8(x-5)} = 8^{\frac{2}{3}}$

$x-5 = 4$; $x = 9$

Check: $9-5 = 4$ ✓

$\log_4(5x+1) = 2$

$4^{\log_4(5x+1)} = 4^2$

$5x+1 = 16$
$5x = 15$
$x = 3$

$5(3)+1 = 16$ ✓

You try ☺
$\log_{16}(x+12) = \frac{3}{2}$

$\log_2(x+3) = 4$

Property of Inequality for Logarithmic Functions:

Algebra: *if* $b > 1, x > 0,$ *and* $\log_b x > y,$ *then* $x > b^y$ (one inequality)

Example: $\log_2 2x > 2$

$2x > 2^2$

$2x > 4$

$x > 2$

$\log_2(8x + 5) > \log_2(6x + 19)$

$8x + 5 > 6x + 19$

$2x > 14$

$x > 7$

You try ☺

$\log_4 8x > 3$

$\log_3(8x + 3) > \log_3(5x + 12)$

Algebra: *if* $b > 1, x > 0,$ *and* $\log_b x < y,$ *then* $0 < x < b^y$ (two inequalities)

Example: $\log_2(3x + 1) < 4$

$3x + 1 < 2^4$

$3x + 1 < 16$

$3x < 15$

$x < 5$

also

$3x + 1 > 0$

$3x > -1$

$x > -1/3$

$-1/3 < x < 5$

$\log_3(3x - 4) < \log_3(x + 1)$

$3x - 4 < x + 1$

$2x < 5$

$x < 5/2$

also

$3x - 4 > 0$

$3x > 4$

$x > 4/3$

$x + 1 > 0$

$x > -1$

$4/3 < x < 5/2$

You try ☺

$\log_3(x + 3) < 3$

$\log_2(3x - 4) < \log_2(2x + 7)$