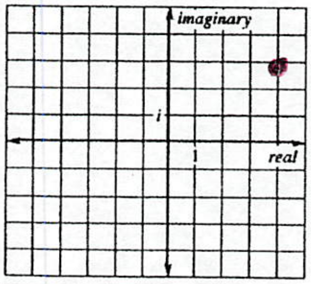
Notes 5.2 Trigonometric Form of Complex Numbers

Graph a Complex Number
Absolute Value of a Complex Number
Trigonometric Form of a Complex Number
Product and Quotient of Complex Numbers in Trigonometric Form

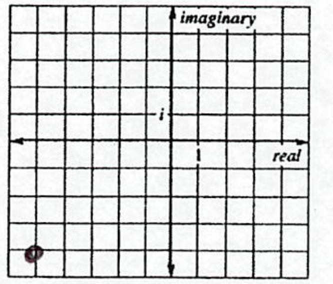
Graph of a Complex Number:

Plot the complex numbers in the same complex plane:

$4+3i$



$-5-4i$

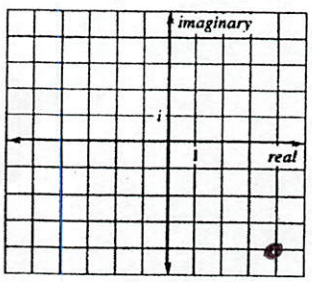


ABSOLUTE VALUE OF A COMPLEX NUMBER

The absolute value of a complex number $z = a + bi$, denoted $|z|$, is a non-negative **real number defined as $|z| = \sqrt{a^2 + b^2}$. This is the distance of z from the** origin **in the complex plane.**

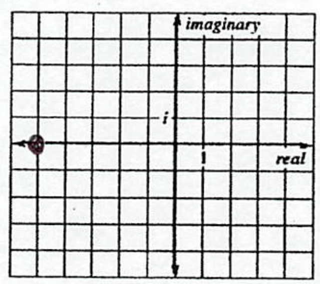
In Exercises 1 to 8, graph each complex number.
Find the absolute value of each complex numbers:

2. $z = 4 - 4i$

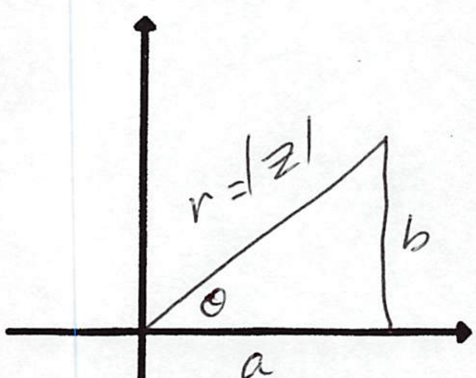


$|z| = \sqrt{4^2 + (-4)^2}$
$= \sqrt{32}$
$= 4\sqrt{2}$

6. $z = -5$



$|z| = 5$



$\cos\theta = \frac{a}{r}$ $\sin\theta = \frac{b}{r}$

$a = r\cos\theta$ $b = r\sin\theta$

Trigonometric Form of a Complex Number:

The ***trigonometric form*** of the complex number ***z = a + bi*** is

$z = r\cos\theta + i\, r\sin\theta$
$= r(\cos\theta + i\sin\theta)$
$= r\, \text{cis}\,\theta$

where :

a = $r\cos\theta$
b = $r\sin\theta$
r = $\sqrt{a^2+b^2}$
tan θ = $\left|\frac{b}{a}\right|$

$\theta = \tan^{-1}\left|\frac{b}{a}\right|$

It's in 3rd quadrant

In Exercises 9 to 16, Write each complex number in trigonometric form.

10. $z = -4 - 4i$

$r = \sqrt{(-4)^2+(-4)^2} = 4\sqrt{2}$

$\theta = \tan^{-1}(1) = 225°$

$4\sqrt{2}\ cis\ 225°$

12. $z = 1 + i\sqrt{3}$

14. $z = -2i$

$r = 2$

$\theta = 270°$

$2\ cis\ 270°$

16. $z = 3$

In Exercises 17 to 34, write each complex number in standard form.

18. $z = 3(\cos 240° + i\ \sin 240°)$

$3\left(-\frac{1}{2} + i\left(\frac{\sqrt{3}}{2}\right)\right)$

$= -3/2 + \frac{3i\sqrt{3}}{2}$

24. $z = 5\ cis\ 90°$

32. $z = cis\ \frac{3\pi}{2}$

$\cos\frac{3\pi}{2} + i\sin\frac{3\pi}{2} = -i$

21. $z = 6\ cis\ 135°$

Product and Quotient of Complex Numbers:

Let: $\boldsymbol{z_1 = r_1(\cos\theta_1 + i\sin\theta_1)}$ *and* $\boldsymbol{z_2 = r_2(\cos\theta_2 + i\sin\theta_2)}$

Then:

$z_1 z_2 = r_1(\cos\theta_1 + i\sin\theta_1)\cdot r_2(\cos\theta_2 + i\sin\theta_2) = r_1r_2(\cos(\theta_1+\theta_2) + i\sin(\theta_1+\theta_2))$

or $r_1r_2\ cis\ (\theta_1+\theta_2)$

$\frac{z_1}{z_2} = \frac{r_1(\cos\theta_1 + i\sin\theta_1)}{r_2(\cos\theta_2 + i\sin\theta_2)} = \frac{r_1}{r_2}(\cos(\theta_1-\theta_2) + i\sin(\theta_1-\theta_2))$

or $\frac{r_1}{r_2}\ cis\ (\theta_1-\theta_2)$

In Exercises 35 to 42, multiply the complex numbers. Write the answer in trigonometric form.

38. $8(\cos 88° + i\sin 88°)\cdot 12(\cos 112° + i\sin 112°)$ $= 8\cdot 12\cos(88+112)$

$= 96\ \cos 200°$

42. $7\ cis\ 0.88 \cdot 5\ cis\ 1.32$

$= 7\cdot 5\ cis\ (.88+1.32) = 35\ cis$

In Exercises 43 to 50, divide the complex numbers. Write the answer in standard form.

44. $\frac{15\ cis\ 240°}{3\ cis\ 135°} = \frac{15}{3}\ cis\ (240-135) =$ ~~$5\ cis\ 105$~~

$= -1.294 + 4.829i$

50. $\frac{18(\cos 0.56 + i\sin 0.56)}{6(\cos 1.22 + i\sin 1.22)} = \frac{18}{6}\ cis\ (-.66)$

$=$ ~~[illegible]~~ $= 2.369 - 1.839i$

In Exercises 51 to 58, Perform the indicated operation in trigonometric form. Write the solution in standard form.

51. $(1 - i\sqrt{3})(1 + i)$

$(2\ cis\ 300)(\sqrt{2}\ cis\ 45)$

$= 2\sqrt{2}\ cis\ 345 = 2.732 - 0.732i$

55. $\frac{(1+i\sqrt{3})}{(1-i\sqrt{3})} = \frac{2\ cis\ 60}{2\ cis\ 300}$

$= cis\ (-240)$

$= cis\ 120 = -\frac{1}{2} + \frac{\sqrt{3}}{2}i$