3.3 Notes- Optimization with Linear Programming

Linear programming:

Feasible region:

Bounded:

Unbounded:

Example 1:

a) Graph the system of inequalities. Name the coordinates of the vertices of the feasible region.

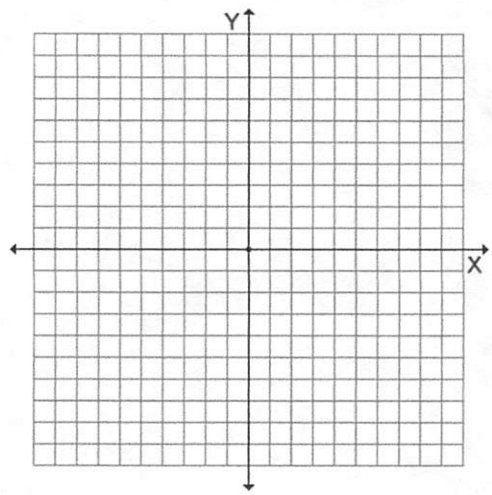
b) Find the maximum and minimum values of the function $f(x, y) = 3x + 2y$ for this polygonal region.

$y \leq 4$

$y \leq -x + 6$

$y \geq \frac{1}{2}x - \frac{3}{2}$

$y \leq 6x + 4$



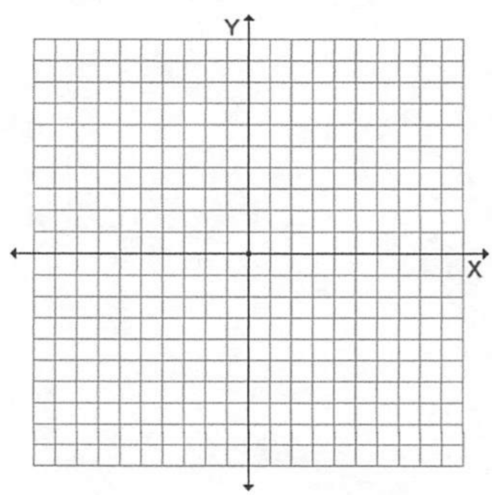
(x, y)	$3x + 2y$	$f(x, y)$

Example 2:

a) Graph the system of inequalities. Name the coordinates of the vertices of the feasible region.

b) Find the maximum and minimum values of the function $f(x, y) = 9x - 6y$ for this polygonal region.

$$2y + 3x \geq -12$$
$$y \leq 3x + 12$$
$$y \geq 3x - 6$$



(x, y)	$9x - 6y$	$f(x, y)$

Solving optimization word problems

A procedure to solve linear programming word problems is illustrated below.

Note how each phrase and number is translated into linear equations and inequalities.

Then, the inequalities are graphed to show the feasibility region.

And, finally, each corner point is tested in the objective function to determine which variables achieve the best outcome.

4 basic steps:

1) Identify and label *variables*

2) Determine the *objective function*

3) List and Graph the *constraints*

4) Test *corner points* of feasibility region

A math test consists of number problems and graphing problems. Number problems are worth 6 points each, and graphing problems are worth 10 points each. You can accurately solve a number problem in 2 minutes and a graphing problem in 4 minutes. Assuming you have 40 minutes and may choose no more than 12 problems to answer, how many of each type should you solve to get the highest score?

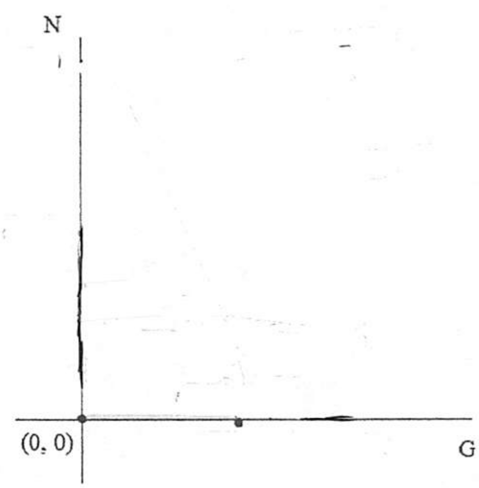
1) Identify and label variables: N = # of number problems G = # of graphing problems

2) Determine the objective function: "how many to get highest score?"

3) List and graph the constraints: (time)

(problems)

4) Test the corner points of the feasibility region



3.3 Notes- Optimization with Linear Programming

Linear programming: Modeling technique to find maximum or minimum of a function

Feasible region:

The vertices of the solution set.

Bounded:

feasible region is enclosed

Unbounded:

feasible region is open

Example 1:

a) Graph the system of inequalities. Name the coordinates of the vertices of the feasible region.

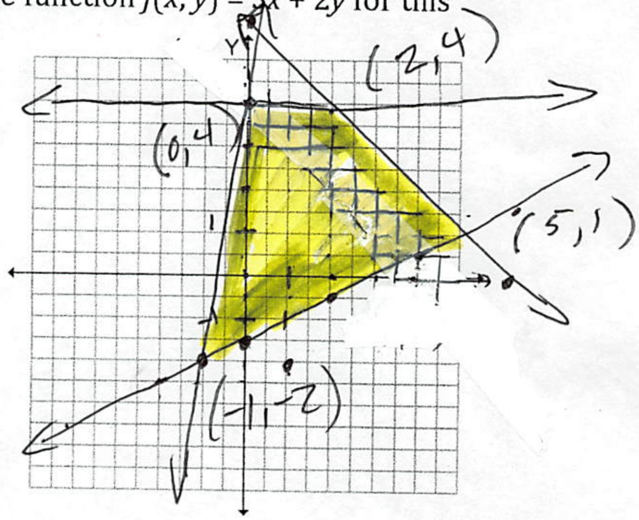
b) Find the maximum and minimum values of the function $f(x, y) = 3x + 2y$ for this polygonal region.

$y \le 4$

$y \le -x + 6$

$y \ge \frac{1}{2}x - \frac{3}{2}$

$y \le 6x + 4$



(x, y)	3x + 2y	f (x, y)
(0,4)	3(0)+2(4)	8
(2,4)	3(2)+2(4)	14
(-1,-2)	3(-1)+2(-2)	-7
(5,1)	3(5)+2(1)	17

min

max

Example 2:

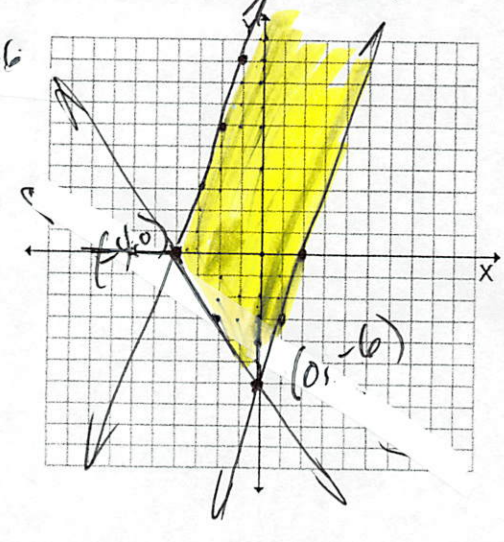
a) Graph the system of inequalities. Name the coordinates of the vertices of the feasible region.

b) Find the maximum and minimum values of the function $f(x, y) = 9x - 6y$ for this polygonal region.

$2y + 3x \ge -12$ $\quad y \ge -\frac{3}{2}x - 6$

$y \le 3x + 12$

$y \ge 3x - 6$



(x, y)	9x - 6y	f (x, y)
(-4,0)	9(-4)-6(0)	-36
(0,-6)	9(0)-6(-6)	36

minimum

maximum

Solving optimization word problems

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1) Identify and label variables: N = # of number problems G = # of graphing problems

2) Determine the objective function: "how many to get highest score?"

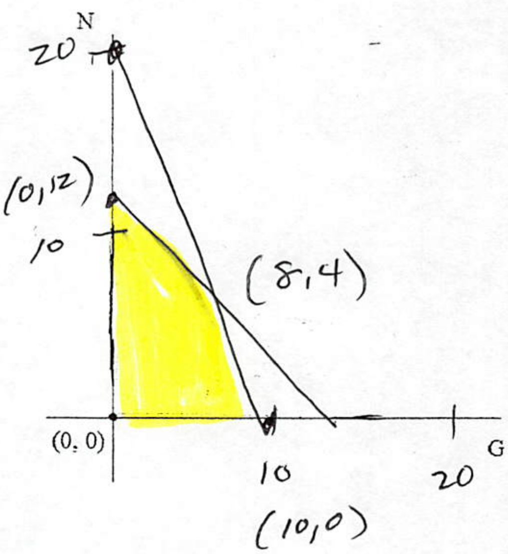
$$\text{Score} = 6n + 10g$$

3) List and graph the constraints: (time) $2n + 4g \le 40 \Rightarrow n \le -2g + 20$

(problems) $n + g \le 12 \Rightarrow n \le -g + 12$

4) Test the corner points of the feasibility region

also
$n \geq 0$
$g \geq 0$
questions has to be positive



$$\text{Score} = 10g + 6n$$

$(10,0) = 10(10) + 6(0) = 100$

$(8,4) = 10(8) + 6(4) = \boxed{104}$

$(0,12) = 10(0) + 6(12) = 72$

Max Score is 104
8 graphing and 4 number