Trig Identities Worksheet 3.1_ Name: _____

1. $\frac{1}{tanx} + tanx = \frac{1}{sinx\,cosx}$

$\cot x + \tan x$

$\frac{\cos x}{\sin x} + \frac{\sin x}{\cos x} = \frac{\cos^2 x + \sin^2 x}{\sin x \cos x}$

$= \frac{1}{\sin x \cos x}$

2. $\frac{\sec\theta \sin\theta}{\tan\theta + \cot\theta} = \sin^2\theta$

$\frac{\frac{1}{\cos\theta}\cdot\sin\theta}{\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}} = \frac{\frac{\sin\theta}{\cos\theta}}{\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}} \cdot \frac{\sin\theta\cos\theta}{\sin\theta\cos\theta}$

$= \frac{\sin^2\theta}{\sin^2\theta + \cos^2\theta} = \frac{\sin^2\theta}{1} = \sin^2\theta$

3. $\frac{\cos x}{1-\sin x} - \frac{\cos x}{1+\sin x} = 2\tan x$

$\frac{\cos x(1+\sin x) - \cos x(1-\sin x)}{(1-\sin x)(1+\sin x)}$

$\frac{\cos x + \cos x\sin x - \cos x + \cos x\sin x}{1-\sin^2 x}$

$= \frac{2\cos x \sin x}{\cos^2 x} = 2\frac{\sin x}{\cos x} = 2\tan x$

4. $\cos^2 x = \frac{\csc x \cos x}{\tan x + \cot x}$

$= \frac{\frac{1}{\sin x}\cdot\cos x}{\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x}} \cdot \frac{\sin x\cos x}{\sin x\cos x}$

$= \frac{\cos^2 x}{\sin^2 x + \cos^2 x} = \frac{\cos^2 x}{1} = \cos^2 x$

5. $\frac{\sin^4 x - \cos^4 x}{\sin^2 x - \cos^2 x} = 1$

$\frac{(\sin^2 x - \cos^2 x)(\sin^2 x + \cos^2 x)}{(\sin^2 x - \cos^2 x)}$

$= 1$

6. $\frac{1-\sin x}{\cos x} = \frac{\cos x}{1+\sin x}$ $\frac{1-\sin x}{(1-\sin x)}$

$= \frac{\cos(1-\sin x)}{1-\sin^2 x} = \frac{\cos x(1-\sin x)}{\cos^2 x}$

$= \frac{1-\sin x}{\cos x}$

7. $1 - 2\cos^2 x = \frac{\tan^2 x - 1}{\tan^2 x + 1}$

$= \frac{\tan^2 x - 1}{\sec^2 x} = \frac{\tan^2 x}{\sec^2 x} - \frac{1}{\sec^2 x}$

$\sin^2 x - \cos^2 x$ $= \frac{\sin^2 x}{\cos^2 x} - \cos^2 x$

$1 - \cos^2 x - \cos^2 x$ $\Leftarrow \frac{\cos^2 x}{1/\cos^2 x}$

$= 1 - 2\cos^2 x$

8. $\tan^2\beta = \csc^2\beta\tan^2\beta - 1$

$\frac{1}{\sin^2\beta}\cdot\frac{\sin^2\beta}{\cos^2\beta} - 1$

$= \frac{1}{\cos^2\beta} - 1$

$= \sec^2\beta - 1$

$= \tan^2\beta$

9. $\sec x + \tan x = \frac{\cos x}{1-\sin x}$ $\frac{1+\sin x}{(1+\sin x)}$

$= \frac{\cos x(1+\sin x)}{1-\sin^2 x}$

$= \frac{\cos x(1+\sin x)}{\cos^2 x}$

$= \frac{1+\sin x}{\cos x}$

$= \frac{1}{\cos x} + \frac{\sin x}{\cos x} = \sec x + \tan x$

10. $\frac{\csc\lambda}{\sin\lambda} - \frac{\cot\lambda}{\tan\lambda} = 1$

$\frac{\frac{1}{\sin\lambda}}{\sin\lambda} - \frac{\frac{\cos\lambda}{\sin\lambda}}{\frac{\sin\lambda}{\cos\lambda}} = \frac{1}{\sin^2\lambda} - \frac{\cos^2\lambda}{\sin^2\lambda}$

$= \frac{1-\cos^2\lambda}{\sin^2\lambda}$

$= \frac{\sin^2\lambda}{\sin^2\lambda} = 1$

11. $\frac{\cos x}{1-\sin x} - \tan x = \sec x$

$\frac{\cos x}{1-\sin x} - \frac{\sin x}{\cos x} = \frac{\cos^2 x - \sin x(1-\sin x)}{(1-\sin x)\cos x}$

$= \frac{\cos^2 x + \sin^2 x - \sin x}{(1-\sin x)\cos x} = \frac{(1-\sin x)}{(1-\sin x)\cos x}$

$= \frac{1}{\cos x} = \sec x$

12. $\sec\theta - \tan\theta \sin\theta = \frac{1}{\sec\theta}$

$\frac{1}{\cos\theta} - \frac{\sin\theta}{\cos\theta} \cdot \sin\theta$

$\frac{1-\sin^2\theta}{\cos\theta} = \frac{\cos^2\theta}{\cos\theta}$

$= \frac{\cos\theta}{\cos\theta} = \frac{1}{\sec\theta}$

13. $\frac{\sin^3 x + \cos^3 x}{\sin x + \cos x} = 1 - \sin x \cos x$

$\frac{(\sin x + \cos x)(\sin^2 x - \sin x\cos x + \cos^2 x)}{(\sin x + \cos x)}$

$= 1 - \sin x \cos x$

14. $\frac{\sin^2\beta}{\cos^2\beta + 3\cos\beta + 2} = \frac{1-\cos\beta}{2+\cos\beta}$

$\frac{\sin^2\beta}{(\cos\beta+2)(\cos\beta+1)} = \frac{1-\cos^2\beta}{(\cos\beta+2)(\cos\beta+1)}$

$\frac{(1+\cos\beta)(1-\cos\beta)}{(\cos\beta+2)(\cos\beta+1)} = \frac{1-\cos\beta}{2+\cos\beta}$

15. $\frac{\cos x + 1}{\sin^3 x} = \frac{\csc x}{1-\cos x}$

$= \frac{\cos x + 1}{\sin x(\sin^2 x)} = \frac{\cos x + 1}{\sin x(1-\cos^2 x)}$

$= \frac{\cos x + 1}{\sin x(1+\cos x)(1-\cos x)} = \frac{1}{\sin x(1-\cos x)}$

$= \frac{\csc x}{1-\cos x}$

16. $\frac{\tan\theta}{\sec\theta} + \frac{\cot\theta}{\csc\theta} = \sin\theta + \cos\theta$

$\frac{\frac{\sin\theta}{\cos\theta}}{1/\cos\theta} + \frac{\frac{\cos\theta}{\sin\theta}}{1/\sin\theta} = \sin\theta + \cos\theta$

17. $\csc^4 x - \cot^4 x = \csc^2 x + \cot^2 x$

$(\csc^2 x - \cot^2 x)(\csc^2 x + \cot^2 x)$

$(1)(\csc^2 x + \cot^2 x)$

$= (\csc^2 x + \cot^2 x)$

18. $\frac{1-\cos x}{\sin x} + \frac{\sin x}{1-\cos x} = 2\csc x$

$\frac{(1-\cos x)^2 + \sin^2 x}{\sin x(1-\cos x)} = \frac{1 - 2\cos x + \cos^2 x + \sin^2 x}{\sin x(1-\cos x)}$

$= \frac{2-2\cos x}{\sin x(1-\cos x)} = \frac{2(1-\cos x)}{\sin x(1-\cos x)} = \frac{2}{\sin x} = 2\csc x$

19. $\frac{1}{1-\sin x} - \frac{1}{1+\sin x} = 2\tan x \sec x$

$\frac{1+\sin x - (1-\sin x)}{(1-\sin x)(1+\sin x)} = \frac{1+\sin x - 1 + \sin x}{1-\sin^2 x}$

$= \frac{2\sin x}{\cos^2 x} = 2\tan x \sec x$

20. $\frac{1+\tan x}{1-\tan x} = \frac{\cos x + \sin x}{\cos x - \sin x}$

$= \frac{1 + \frac{\sin x}{\cos x}}{1 - \frac{\sin x}{\cos x}} \cdot \frac{\cos x}{\cos x} = \frac{\cos x + \sin x}{\cos x - \sin x}$

Trig Identities worksheet name:

Prove each identity:

1. $\sec x - \tan x \sin x = \frac{1}{\sec x}$

$\frac{1}{\cos x} - \frac{\sin x}{\cos x} \cdot \sin x$

$\frac{1 - \sin^2 x}{\cos x} = \frac{\cos^2 x}{\cos x} = \cos x$

2. $\frac{1+\cos x}{\sin x} = \csc x + \cot x$

$\frac{1}{\sin x} + \frac{\cos x}{\sin x}$

$= \frac{1+\cos x}{\sin x}$

3. $\frac{\sec\theta \sin\theta}{\tan\theta + \cot\theta} = \sin^2\theta$

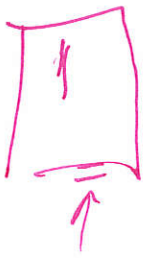
$\frac{\frac{1}{\cos\theta} \cdot \sin\theta}{\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}} = \frac{\frac{\sin\theta}{\cos\theta}}{\frac{\sin^2\theta + \cos^2\theta}{\cos\theta \cdot \sin\theta}} = 1$

$= \frac{\sin\theta}{\cos\theta} \cdot \frac{\cos\theta\sin\theta}{1} = \sin^2\theta$

4. $\frac{\sec\theta}{\cos\theta} - \frac{\tan\theta}{\cot\theta} = 1$

$\frac{\frac{1}{\cos\theta}}{\cos\theta} - \frac{\frac{\sin\theta}{\cos\theta}}{\frac{\cos\theta}{\sin\theta}}$

$\frac{1}{\cos^2\theta} - \frac{\sin^2\theta}{\cos^2\theta} = \frac{1-\sin^2\theta}{\cos^2\theta} = \frac{\cos^2\theta}{\cos^2\theta}$



5. $\cos^2 y - \sin^2 y = 1 - 2\sin^2 y$

$(1-\sin^2 y) - \sin^2 y$

$= 1 - 2\sin^2 y$

6. $\csc^2\theta \tan^2\theta - 1 = \tan^2\theta$

$\frac{1}{\sin^2\theta} \cdot \frac{\sin^2\theta}{\cos^2\theta} - 1$

$\frac{1}{\cos^2\theta} - 1$

$\sec^2\theta - 1 = \tan^2\theta$

7. $\frac{\sec^2\theta}{\sec^2\theta - 1} = \csc^2\theta$

$\frac{\sec^2\theta}{\tan^2\theta}$

$\frac{\frac{1}{\cos^2\theta}}{\frac{\sin^2\theta}{\cos^2\theta}} = \frac{1}{\cos^2\theta} \cdot \frac{\cos^2\theta}{\sin^2\theta} = \frac{1}{\sin^2\theta}$

$= \csc^2\theta$

8. $\tan^2 x \sin^2 x = \tan^2 x - \sin^2 x$

$\tan^2 x (1 - \cos^2 x)$

$\tan^2 x - \tan^2 x \cdot \cos^2 x$

$\frac{\sin^2 x}{\cos^2 x} \cdot \cos^2 x$

$\tan^2 x - \sin^2 x$

Trig Identities worksheet

9. $(\sin\theta+\cos\theta)^2+(\sin\theta-\cos\theta)^2=2$

$\sin^2\theta+2\sin\theta\cos\theta+\cos^2\theta+\sin^2\theta+\cos^2\theta$
$-2\sin\theta\cos\theta$

$1+\quad\cdots\cdots\cdots\cdot\ \sin^2\theta+\cos^2\theta$

$1+1=2$

10. $(\sin\theta+\cos\theta)(\tan\theta+\cot\theta)=\sec\theta+\csc\theta$

$\frac{\sin\theta}{\cos\theta}+\frac{\cos\theta}{\sin\theta}$

$\frac{\sin^2\theta}{\cos\theta}+\cos\theta+\sin\theta+\frac{\cos^2\theta}{\sin\theta}$

$\frac{\sin^2\theta}{\cos\theta}+\frac{\cos^2\theta}{\cos\theta}+\frac{\sin^2\theta}{\sin\theta}+\frac{\cos^2\theta}{\sin\theta}$

$\frac{\sin^2+\cos^2\theta}{\cos\theta}+\frac{\sin^2\theta+\cos^2\theta}{\sin\theta}$

$=\frac{1}{\cos\theta}+\frac{1}{\sin\theta}=\sec\theta+\csc\theta$

11. $\frac{\tan\theta-1}{\tan\theta+1}=\frac{1-\cot\theta}{1+\cot\theta}$

$\frac{\frac{\sin\theta}{\cos\theta}-1}{\frac{\sin\theta}{\cos\theta}+1}=\frac{\frac{\sin\theta-\cos\theta}{\cos\theta}}{\frac{\sin\theta+\cos\theta}{\cos\theta}}$

$=\frac{\sin\theta-\cos\theta}{\sin\theta+\cos\theta}\cdot\frac{\frac{1}{\sin\theta}}{\frac{1}{\sin\theta}}=\frac{1-\cot\theta}{1+\cot\theta}$

12. $\frac{1-\tan^2x}{1+\tan^2x}=1-2\sin^2x$

$\frac{1-\tan^2x}{\sec^2x}=\frac{1-\tan^2x}{\frac{1}{\cos^2x}}=(1-\tan^2x)\cos^2x$

$=\cos^2x-\tan^2x\cos^2x=\cos^2x-\frac{\sin^2x}{\cos^2x}\cdot\cos^2x$

$=\cos^2x-\sin^2x$

$=(1-\sin^2x)-\sin^2x=1-2\sin^2x$

13. $\frac{\cos x+1}{\sin^3x}=\frac{\csc x}{1-\cos x}$

$=\frac{\cos x+1}{\sin^2x\cdot\sin x}=\frac{\cos x+1}{(1-\cos^2x)\sin x}$

$=\frac{(\cos x+1)}{(1+\cos x)(1-\cos x)\sin x}=\frac{1}{(1-\cos x)(\sin x)}$

$=\frac{1}{1-\cos x}\cdot\frac{\csc x}{1}=\frac{\csc x}{1-\cos x}$

14. $\csc^4x-\cot^4x=\csc^2x+\cot^2x$

$(\csc^2x-\cot^2x)(\csc^2x+\cot^2x)$

$1\ (\csc^2x+\cot^2x)$

$=\csc^2x+\cot^2x$

15. $\frac{\tan\theta}{\sec\theta}+\frac{\cot\theta}{\csc\theta}=\sin\theta+\cos\theta$

$\frac{\frac{\sin\theta}{\cos\theta}}{\frac{1}{\cos\theta}}+\frac{\frac{\cos\theta}{\sin\theta}}{\frac{1}{\sin\theta}}$

$=\sin\theta+\cos\theta$

16. $\frac{\sin y+\tan y}{1+\sec y}=\sin y$

$\frac{\sin y+\frac{\sin y}{\cos y}}{1+\frac{1}{\cos y}}=\frac{\frac{\sin y\cos y}{\cos y}+\frac{\sin y}{\cos y}}{\frac{\cos y}{\cos y}+\frac{1}{\cos y}}$

$=\frac{\sin y\cos y+\sin y}{\cos y}\cdot\frac{\cos y}{\cos y+1}$

$\frac{\sin y\cos y+\sin y}{\cos y+1}=\frac{\sin y(\cos y+1)}{(\cos y+1)}$